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केन्द्रीय विद्यालय संगठन

CLASS: XII

MATHEMATICS

HIGHER ORDER THINKING SKILLS

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KENDRIYA VIDYALAYA SANGATHAN

BANGALORE REGION

QUESTION BANK
HIGHER ORDER THINKING SKILLS
CLASS XII - MATHEMATICS

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RELATIONS AND FUNCTIONS

1 Mark

1) If $f(x) = \frac{x-1}{x+1}, (x \neq 1,-1)$, show that fof^{-1} is an identity function.

2) If $f(x) = Sin^2x + Sin^2\left(x+\frac{\Pi}{3}\right) + Cosx.Cos\left(x+\frac{\Pi}{3}\right)$ and $g\left(\frac{5}{4}\right) = 1$, then find the value of gof(x) = x

(Ans : 1)

3) Let * be a binary operation on the set of real numbers. If a * b = a+b-ab, 2 * (3 * x) = -7, find the value of x. (Ans : x=-2)

4) Find the number of One-One functions from a finite set A to A, where n(A) = P (Ans : P!)

5) Let A = {4,5,0}. Find the number of binary operations that can be defined on A. (Ans: 3^9)

6) Let $\int:\{R \to R\}$ be a function defined by $f(x) = x^2-3x+4$, for all $x \, \varepsilon \, R$, find the value of $f^{-1}(2)$

Ans: {1,2}

7) Let $f: R \to R$ defined by $f(x) = (ax^2+b)^3$ find the function $g: R \to R$ such that f(g(x)) = g(f(x))

Ans: $\left[\frac{x^{\frac{1}{3}}-b}{a}\right]^{\frac{1}{2}}$

8) If $f(x) = \frac{5x+3}{4x-5}\left(x \neq \frac{5}{4}\right)$, find g(x) such that gof(x) = x Ans: $g(x) = \frac{5x+3}{4x-5}$

9) If $f(x) = \frac{1+x}{1-x}$, show that $f[f(\tan\theta)] = -\cot\theta$

10) Show that $\frac{1}{\sin^3 x} + \cot x + \frac{1}{x^5} + x^3$ is an odd function.

11) Let f, g be two functions defined by

$f(x) = \frac{x}{x+1}, g(x) = \frac{x}{1-x}$, then find $(fog)^{-1}(x)$ Ans: x

12) Let $f(x) = \frac{\alpha x}{x+1}$, $x \neq -1$, find the value of α such that f(f(x)) = x Ans: $\alpha = -1$

4 Marks / 6 marks

13) If $f(x) = \log\left(\frac{1+x}{1-x}\right)$, show that $f(x) + f(y) = f\left(\frac{x+y}{1+xy}\right)$

14) If R is a relation on a set $A(A \neq \phi)$, prove that R is symmetric iff R^{-1} = R

15) Show that the relation R on N x N defined by (a,b)R(c,d) $\Leftrightarrow$ a+d = b+c is an equivalence relation.

16) Let $f: R_+ \to [-5,\infty)$ given by $f(x) = 9x^2+6x-5$. Show that f is invertiable with $f^{-1}(y) = \frac{\sqrt{y+6}-1}{3}$

17) Show that the relation "congruence modulo 2" on the set Z (set of integers) is an equivalence relation. Also find the equivalence class of 1.

18) If the function $f : R \rightarrow A$ given by $f(x) = \frac{x^2}{x^2+1}$ is surjection, find the set A.

Ans: A = Range of f(x)=[0,1)

19) Let a relation R on the set R of real numbers be defined as (a,b) $\varepsilon R \Leftrightarrow 1 + ab > 0$ for all a, b ε R. show that R is reflexive and symmetric but not transitive.

20) Let a relation R on the set R of real numbers defined as $(x, y) \varepsilon R \Leftrightarrow x^2 4xy + 3y^2 = 0$. Show that R is reflexive but neither symmetric nor transitive.

21) Let N denote the set of all natural numbers and R be the relation on N x N defined by $(a,b)R(c,d) \Leftrightarrow ad(b+c) = bc(a+d)$. Show that R is an equivalence relation on N x N.

22) Prove that the inverse of an equivalence relation is an equivalence relation.

23) Let $f : A \rightarrow B$ be a given function. A relation R in the set A is given by $R = \{(a,b) \ \varepsilon \ A \ x \ A : f(a) = f(b)\}$. Check, if R is an equivalence relation. Ans: Yes

24) Let f and g be real valued functions, such that (fog)(x) = $\cos x^3$ and (gof)(x) = $\cos^3 x$, find the functions f and g. Ans: f(x)=cosx, g(x) = x^3

25) Define a binary operation * on the set A = {0,1,2,3,4,5} as

$$a * b = \begin{cases} a + b, & \text{if } a + b < 6 \\ a + b - 6, & \text{if } a + b \geq 6 \end{cases}$$

Show that zero is an identity element for this operation and each element a of the set is invertible with 6-a being the inverse of a.

26) Show that the function $f : N \rightarrow N$ given by $f(x) = x - (-1)^x$ for all $x \ \varepsilon \ N$ is a bijection.

27) Prove that relation R defined on the set N of natural numbers by

x R y $\Leftrightarrow$ $2x^2-3xy+y^2 = 0$ is not symmetric but it is reflexive.

28) Let X = {1,2,3,4,5,6,7,8,9}. Let R_1 and R_2 be relations in X given by

R_1 = {(x,y) : x-y is divisible by 3} and

R_2 = {(x,y) ! {x,y} ε {1,4,7} or {x,y} ε {2,5,8}

or {x,y} ε {3,6,9}}. Show that $R_1 = R_2$

29) Determine which of the following functions

f : R $\rightarrow$ R are (a) One - One (b) Onto

(i) f(x) = |x| + x

(ii) f(x) = x - [x]

(Ans: (i) and (ii) $\rightarrow$ Neither One-One nor Onto)

30) On the set N of natural numbers, define the operation * on N by m*n = gcd(m,n) for all m, n ε N. Show that * is commutative as well as associative.

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INVERSE TRIGONOMETRIC FUNCTIONS

1 Mark

1) Find the value of $\cos\left\{\text{Cos}^{-1}\left(\frac{-\sqrt{3}}{2}\right)+\frac{\Pi}{6}\right\}$ Ans: -1

2) Find the value of $\tan^{-1}1 + \tan^{-1}2 + \tan^{-1}3$. Ans: Π

3) Solve for x : $\text{Sin}\left[\text{Sin}^{-1}\frac{1}{5}+\text{Cos}^{-1}x\right]=1$ Ans: $\frac{1}{5}$

4) Write the simplest form : $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)$ Ans: $\frac{\Pi}{4}+\frac{x}{2}$

5) Considering the principal solutions, find the number of solutions of

$\tan^{-1}2x+\tan^{-1}3x=\frac{\Pi}{4}$ Ans: 2

6) Find the principal value of $\text{Sin}^{-1}\left(\frac{-\sqrt{3}}{2}\right)+\text{Cos}^{-1}\left(\frac{-\sqrt{3}}{2}\right)$ Ans: $\frac{\Pi}{2}$

7) Find the value of x if $\text{Cosec}^{-1}x+2\text{Cot}^{-1}7+\text{Cos}^{-1}\frac{3}{4}$ Ans: $x=\text{Cosec}^{-1}\frac{125}{117}$

8) If $\text{Cos}^{-1}x = \tan^{-1}x$, show that $\text{Sin}(\cos^{-1}x) = x^2$

9) If x > 0 and $\text{Sin}^{-1}\left(\frac{5}{x}\right)+\text{Sin}^{-1}\left(\frac{12}{x}\right)=\frac{\Pi}{2}$, then find the value of x. Ans: x = 13

10) Prove that $\text{Cos}\left\{2\text{Cot}^{-1}\sqrt{\frac{1-x}{1+x}}\right\}+x=0$

4 Marks / 6 Marks

11) Prove that $4\tan^{-1}\frac{1}{5}-\tan^{-1}\frac{1}{70}+\tan^{-1}\frac{1}{99}=\frac{\Pi}{4}$

12) If $x, y, z\ \varepsilon\ [-1,1]$ such that $\text{Sin}^{-1}x+\text{Sin}^{-1}y+\text{Sin}^{-1}z=\frac{3\Pi}{2}$, find the value of

$x^{2006}+y^{2007}+z^{2008}-\frac{9}{x^{2006}+y^{2007}+z^{2008}}$ Ans: zero ; x=1,y=1,z=1

13) If $\text{Cos}^{-1}\frac{x}{a}+\text{Cos}^{-1}\frac{y}{b}=\alpha$, prove that $\frac{x^2}{a^2}-\frac{2xy}{ab}\cos\alpha+\frac{y^2}{b^2}=\sin^2\alpha$

14) If $\text{Sin}^{-1}x+\text{Sin}^{-1}y+\text{Sin}^{-1}z=\Pi$, prove that $x\sqrt{1-x^2}+y\sqrt{1-y^2}+z\sqrt{1-z^2}=2xyz$

15) Prove that : $\text{Sin}2\left[\cot^{-1}\left\{\cos(\tan^{-1}x)\right\}\right]=\sqrt{\frac{x^2+1}{x^2+2}}$

16) In any Triangle ABC, if A = $\tan^{-1}2$ and B = $\tan^{-1}3$, prove that $C=\frac{\Pi}{4}$

17) If $x = \operatorname{cosec}\left[\tan^{-1}\left\{\cos(\cot^{-1}(\sec(\sin^{-1} a)))\right\}\right]$ and

$y = \operatorname{Sec}\left[\cot^{-1}\left\{\sin(\tan^{-1}(\operatorname{cosec}(\cos^{-1} a)))\right\}\right]$ where $a \,\varepsilon [0,1]$

Find the relationship between x and y in terms of a Ans: $x^2=y^2=3-a^2$

18) Prove that : $\operatorname{Cot}^{-1}\left[\frac{ab+1}{a-b}\right]+\operatorname{Cot}^{-1}\left[\frac{bc+1}{b-c}\right]+\operatorname{Cot}^{-1}\left[\frac{ca+1}{c-a}\right]=0$

19) Solve for x : $\operatorname{Sin}^{-1}\frac{2\alpha}{1+\alpha^2}+\operatorname{Sin}^{-1}\frac{2\beta}{1+\beta^2}=2\tan^{-1}x$ Ans: $x=\frac{\alpha+\beta}{1-\alpha\beta}$

20) Prove : $\cos^{-1}x-\cos^{-1}y=\cos^{-1}\left[xy+\sqrt{1-x^2}.\sqrt{1-y^2}\right]$

21) If $\tan^{-1}a+\tan^{-1}b+\tan^{-1}c=\Pi$, Prove that a+b+c -= abc

22) Prove that $\tan^{-1}\frac{yz}{xr}+\tan^{-1}\frac{zx}{yr}+\tan^{-1}\frac{xy}{zr}=\frac{\Pi}{2}$ where $x^2+y^2+z^2=r^2$

23) Solve for x : $\tan^{-1}(x+1)+\tan^{-1}x+\tan^{-1}(x-1)=\tan^{-1}3$ Ans: x = -1

24) Solve : $\operatorname{Sin}\left[2\cos^{-1}\left\{\cot(2\tan^{-1}x)\right\}\right]=0$ Ans: $\pm 1, -1\pm\sqrt{2}, 1\pm\sqrt{2}$

25) If $\operatorname{Cos}^{-1}\frac{x}{2}+\cos^{-1}\frac{y}{3}=\theta$, Prove that $9x^2-12xy\cos\theta+4y^2=36\sin^2\theta$

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HIGHLY ORDER THINKING QUESTIONS

HOTS - MATRICES / DETERMINANTS

1) If $a+b+c=0$ and $\begin{vmatrix} a-x & a & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix}=0$ then prove that either x = 0 or $x=\pm\sqrt{\frac{3}{2}\left(a^2+b^2+c^2\right)}$

2) If $A=\begin{bmatrix} p & q \\ r & -p \end{bmatrix}$ is such that $A^2=I$ then find the value of $I-P^2+qr$

3) If $A=\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ and $A+A^T=I$. Find the possible values of θ $\theta=\Pi/3$

4) Inverse of a square matrix is unique. Give an example to prove it?

5) Prove that $\begin{vmatrix} x-3 & x-4 & x-a \\ x-2 & x-3 & x-b \\ x-1 & x-2 & x-c \end{vmatrix}=0$, where a, b, c are in A.P.

6) Using properties of Determinants prove that : $\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+ac & c^2 \end{vmatrix}=4a^2b^2c^2$

7) Express $\begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix}$ as sum of the symmetric and skew symmetric matrices.

8) Prove that $\begin{vmatrix} (b+c)^2 & ab & ca \\ ab & (a+c)^2 & bc \\ ca & bc & (a+b)^2 \end{vmatrix}=2abc(a+b+c)^2$ (Use properties to prove the above)

9) Prove the determinant $\begin{vmatrix} x & -\sin\alpha & \cos\alpha \\ \sin\alpha & -x & 1 \\ \cos\alpha & 1 & x \end{vmatrix}$ is independent as α (Ans: Scalar term)

10) The total number of elements in a matrix represents a prime number. How many possible orders a matrix can have. 2

11) Find the matrix X such that : $\begin{bmatrix} 2 & -1 \\ 0 & 1 \\ -2 & 4 \end{bmatrix}X=\begin{bmatrix} -1 & -8 & -10 \\ 3 & 4 & 0 \\ 10 & 20 & 10 \end{bmatrix}$

12) If $f(x) = 3x^2-9x+7$, then for a square matrix A, write f(A) $(3A^2-9A+7I)$

13) Prove that $\begin{bmatrix} (a+1) & (a+2) & (a+2) & 1 \\ (a+2) & (a+3) & a+3) & 1 \\ (a+3) & (a+4) & (a+4) & 1 \end{bmatrix}=-2$

14) If $\begin{bmatrix} \cos^2 A & \cos A \sin A \\ \cos A \sin A & \sin^2 A \end{bmatrix}$, $Y = \begin{bmatrix} \cos^2 B & \cos B \sin B \\ \cos B \sin B & \sin^2 B \end{bmatrix}$

then show that XY is a zero matrix, provided (A-B) is an odd multiple of $\Pi/2$

15) Give that x = -9 is a root of $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$ find the other roots. Hint: Evaluate, find other roots.

16) If A = $\begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$ find x and y such that $A^2+xI = yA$. Find A^{-1}

17) If P and Q are equal matrices of same order such that PQ = QP, then prove by induction that $PQ^n = Q^nP$. Further, show that $(PQ)^n = P^n \times Q^n$

18) If $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$, then $A^5 = ?$ I_3

19) If A and B are two matrices such that AB = B and BA = A then A^2+B^2 = _____?______\

20) If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ then prove that $A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}$ $n \varepsilon N$

21) Find the values of a, b, c if the matrix

A = $\begin{vmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{vmatrix}$ satisfy the equation $A^I A = I_3$

22) Assume X, Y, Z, W and P are matrices of order (2 x n), (3xk), (n x 3) and (p x k) respectively, then the restriction on n, k and p so that py + my will be defined are : (k=3, p=n)

23) Let A and B be 3 x 3 matrices such that $A^T = -A$ and $B^T = B$. Then the matrix λ(AB+3BA) is skew symmetric matrix for λ. (λ=3)

24) If A = $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2-5A+7I = 0$, use the result to find A^4 $\begin{bmatrix} 39 & 55 \\ -55 & -16 \end{bmatrix}$

25) For what value of 'K' the matrix $\begin{bmatrix} k & 2 \\ 3 & 4 \end{bmatrix}$ has no inverse. (K = 3/2)

26) If A is a non-singular matrix of order 3 and |A| = -4 Find |adj A| (16)

27) Given A = $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ such that | A | = -10. Find $a_{11}c_{11} + a_{12}c_{12}$ (10)

28) If $\begin{vmatrix} x & b & c \\ a & y & c \\ a & b & z \end{vmatrix} = 0$, then find the value of $\frac{x}{x-a} + \frac{y}{y-b} + \frac{z}{z-c}$ (2)

29) If A = $\begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix}$ satisfies the equation :

x^2-6x+17 = 0 find A^{-1} $\quad$ Ans: $\left(\frac{1}{17}\begin{bmatrix} 4 & 3 \\ -3 & 2 \end{bmatrix}\right)$

30) Find the matrix x if $\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} x \begin{bmatrix} 0 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 6 \\ 3 & -2 \end{bmatrix}$

$X = -\frac{1}{4}\begin{bmatrix} -53 & 18 \\ 25 & -10 \end{bmatrix}$

31) If P(a) = $\begin{bmatrix} \cos a & -\sin a & 0 \\ \sin a & \cos a & 0 \\ 0 & 0 & 1 \end{bmatrix}$ then show that $[P(x)]^{-1} = [P(-x)]$

32) If two matrices A^{-1} and B are given how to find $(AB)^{-1}$ verify with an example.

(Find B^{-1} then find B^{-1} x A^{-1})

33) If A = $\begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}$ Verify $(AdjA)^{-1}$ = adj(A^{-1})

34) Find the values of a and b such that A^2 + aI = bA where A = $\begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$ (a=b=8)

35) If $P(\alpha) = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ show that $p(\alpha) \text{ x } p(\beta) = p(\alpha+\beta)$

36) If A = $\begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}$ then prove by Mathematical Induction that : $A^n = \begin{bmatrix} \cos n\theta & i\sin n\theta \\ i\sin n\theta & \cos n\theta \end{bmatrix}$

37) If A = $\begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix}$ find Matrix B such that AB = I, Ans : $B = \begin{bmatrix} 1 & 2 \\ \frac{1}{2} & \frac{3}{2} \end{bmatrix}$

38) If x, y, z are positive and unequal show that the value of determinant $\begin{vmatrix} x & y & z \\ y & z & x \\ z & x & y \end{vmatrix}$ is negative.

39) If $A+B+C=\Pi$, show that $\begin{vmatrix} \sin(A+B+C) & \sin B & \cos C \\ -\sin B & 0 & \tan A \\ \cos(A+B) & -\tan A & 0 \end{vmatrix} = 0$

40) Find the quadratic function defined by the equation f(x) = ax^2+bx+c
if f(o) = 6, f(2) = 11, f(-3) = 6, using determinants.

41) If x, y and z all positive, are p^{th}, q^{th} and r^{th} terms of a G.P. Prove that $\begin{vmatrix} \log x & p & 1 \\ \log y & q & 1 \\ \log z & r & 1 \end{vmatrix} = 0$

42) If a, b, c are in A.P. then find the value of : $\begin{vmatrix} 2y+4 & 5y+7 & 8y+a \\ 3y+5 & 6y+8 & 9y+b \\ 4y+6 & 7y+9 & 10y+c \end{vmatrix}$ (O)

43) If A = $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ then show that $A^n = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$

44) If A = $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ find A^2 Hence find A^6 Ans: $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

45) Find x, if $[x-5-1]\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$ Ans: $x = \pm 4\sqrt{3}$

46) If P and Q are invertible matrices of same order, then show that PQ is also invertible.

47) If the points (2,0), (0,5) and (x,y) are collinear give a relation between x and y.

48) Let $\begin{vmatrix} 3 & y \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$ find the possible values of x and y, find the values if x = y.

49) If A = $\begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$ prove that $A^n = \begin{bmatrix} a^n & \frac{b(a^n-1)}{a-1} \\ 0 & 1 \end{bmatrix}, n \varepsilon N$

50) For any square matrix verify A(adj A) = |A|I

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CONTINUITY AND DIFFERENTIABILITY

1 Mark

1) Let f be continuous function on [1,3]. If f takes only rational values for all x and f(2) = 10 then find the value of f(1.5) Ans: 10

2) Let f be a non zero continuous function satisfying f(x+y) = f(x).f(y) for all x, y ε R. If f(2) = 9 then find the value of f(3).

Ans: 3^3=27, f(x) is of the form a^x.

3) Find $\frac{dy}{dx}$ when $y = x^{x^{x^{\cdots}}}$ Ans: $\frac{dy}{dx} = \frac{y^2}{x(1-y\log x)}$

4) Find the set of all points where the function f(x) = 2x|x| is differentiable. Ans: $(-\alpha, \alpha)$

5) If $f^{\prime}(x)$ = g(x) and $g^{\prime}(x)$ = -f(x) for all n and f(2) = $f^{\prime}(2)$=4, find the value of $f^2(24)+g^2(24)$

Ans: 32

6) If $f(x) = \begin{vmatrix} x & x^2 & x^3 \\ 1 & 2x & 3x^2 \\ 0 & 2 & 6x \end{vmatrix}$ then find $f^{\prime}(x)$ Ans: $6x^2$

7) Let y = $y = \sqrt{e^{\sqrt{x}}}$, $x > 0$ find $\frac{dy}{dx}$ Ans: $\frac{1}{4}\sqrt{\frac{e^{\sqrt{x}}}{x}}$

8) Verify Rolle's theorem for the function f(x) = sin2x in $\left[0, \frac{\Pi}{2}\right]$ Ans: C= $\frac{\Pi}{4}$

9) Find $\frac{dy}{dx}$ when $y = a^x.x^a$ Ans: $a^x.x^{a-1}(a + x\log a)$

10) Find $\frac{dy}{dx}$ when $y = \tan^{-1}\frac{1+x^2}{1-x^2}$ Ans: $\frac{2x}{1+x^4}$

4 MARKS/6 MARKS

11) Given that $f(x) = \begin{cases} \frac{1-\cos 4x}{x^2}, & \text{if } x < 0 \\ a & \text{, if } x = 0 \\ \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4} & \text{, if } x > 0 \end{cases}$

If f(x) is continuous at x = 0, find the value of a. Ans: a=8

12) Show that the function : $f(x) = \begin{cases} \frac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}, & \text{when } x \neq 0 \\ 0\ , & \text{when } x = 0 \end{cases}$ is discontinuous at x = 0

13) Is the function $f(x) = \frac{3x + 4\tan x}{x}$ continuous at x = 0? If not, how may the function be defined to make it continuous at this point.

Ans: $f(x) = \begin{cases} \frac{3x+4\tan x}{x}, & \text{when } x \neq 0 \\ 7\ , & \text{when } x = 0 \end{cases}$

14) Find a and b if the function :

$$f(x)=\begin{cases}[1+|\sin x|]^{\frac{a}{|\sin x|}} & , \quad \frac{-\Pi}{6}<x<0 \\ 6 & , \quad x=0 \\ e^{\frac{\tan 2x}{\tan 3x}} & , \quad 0<x<\frac{\Pi}{6}\end{cases}$$ is continuous on $\left(\frac{-\Pi}{6},\frac{\Pi}{6}\right)$

15) Show that the function : $f(x)=\begin{cases}|x|, & x\le 2\\ [x], & x>2\end{cases}$ is continuous on $[0,2]$

16) Show that Sin|x| is continuous.

17) Show that the function $f(x)=\begin{cases}x+\lambda, & x<1\\ \lambda x^2+1, & x\ge 1\end{cases}$

is continuous function, regardless of the choice of $\lambda \varepsilon R$

18) Determine the values of a, b and c for which the function :

$$f(x)=\begin{cases}\frac{\sin(a+1)x+\sin x}{x} & , \quad x<0\\ c & , \quad x=0\\ \frac{\sqrt{x+bx^2}-\sqrt{x}}{b\sqrt{x^3}} & , \quad x>0\end{cases}$$ may continuous at x = 0

Ans: $a=-\frac{3}{2}, c=\frac{1}{2}$, b may have any real number.

19) Show that the function f(x) = |Sinx+cosx| is continuous at x = Π

20) The function $f(x)=\frac{\log(1+ax)-\log(1-bx)}{x}$ is not defined at x = 0. Find the value of f(x) so that f(x) is continuous at x = 0.

Ans: f(x) to be continuous at x = 0, f(o) = a+b

21) Find all the points of discontinuity of f defined by f(x) = |x|-|x+1| Ans: x=0, -1

22) Let $f(x)=\begin{cases}|x|\cos\frac{1}{x} & \text{if } x\neq 0\\ 0 & \text{if } x=0\end{cases}$ then discuss the continuity of f(x) at x = 0

Ans: Yes, f(x) is continuous at x = 0

23) Discuss the continuity of the following function at x = 0

$$f(x)=\begin{cases}\frac{x^4+x^3+2x^2}{\tan^{-1}x}, & x\neq 0\\ 10 \ , & x=0\end{cases}$$

24) Let $f(x)=\begin{cases}\dfrac{1-\sin^3 x}{3\cos^2 x} & \text{if } x<\dfrac{\Pi}{2}\\ a & \text{if } x=\dfrac{\Pi}{2}\\ \dfrac{b(1-\sin x)}{(\Pi-2x)^2} & \text{if } x>\dfrac{\Pi}{2}\end{cases}$ If f(x) is continuous at x $x=\dfrac{\Pi}{2}$, find a and b.

Ans : a=½, b = 4

25) Let $\phi(x)$ and $\varphi(x)$ be derivable at x = 3. Show that necessary and sufficient condition for the function defined as :

$f(x)=\begin{cases}\phi(x), & x\le c\\ \varphi(x), & x>c\end{cases}$ to be derivable at $x=c$ are (i) $\phi(c)=\varphi(c)$ (ii) $\phi^{|}(c)=\varphi^{|}(c)$

26) Find $\dfrac{dy}{dx}$ when $y=\text{Sin}^{-1}\left[\dfrac{5x+12\sqrt{1-x^2}}{13}\right]$ Ans: $\dfrac{1}{\sqrt{1-x^2}}$

27) If $y=\text{Sin}^{-1}\left[x^2\sqrt{1-x^2}+x\sqrt{1-x^4}\right]$, prove that $\dfrac{dy}{dx}=\dfrac{2x}{\sqrt{1-x^4}}+\dfrac{1}{\sqrt{1-x^2}}$

28) If $y=\dfrac{2}{\sqrt{a^2-b^2}}\tan^{-1}\left[\sqrt{\dfrac{a-b}{a+b}}\tan\dfrac{x}{2}\right]$, prove that $\dfrac{dy}{dx}=\dfrac{1}{a+b\cos x}, a>b>0$

29) Differentiate w.r.t. x, $y=\tan^{-1}\left[\dfrac{a\cos x-b\sin x}{b\cos x+a\sin x}\right]$ Ans: -1

30) Find $\dfrac{dy}{dx}$, when $y=\sin^{-1}\left[x\sqrt{x-1}-\sqrt{x}\sqrt{1-x^2}\right]$ Ans: $\dfrac{1}{\sqrt{1-x^2}}-\dfrac{1}{2\sqrt{x}\sqrt{1-x^2}}$

31) Given that $\text{Cos}\dfrac{x}{2}.\text{Cos}\dfrac{x}{4}.\text{Cos}\dfrac{x}{8}.........=\dfrac{\text{Sin}x}{x}$,

Prove that $\dfrac{1}{2^2}\text{Sec}^2\dfrac{x}{2}+\dfrac{1}{2^4}\text{Sec}^2\dfrac{x}{4}+.......=\text{Cosec}^2 x-\dfrac{1}{x^2}$

32) Let $y=\text{Cot}^{-1}\left[\dfrac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]$, find $\dfrac{dy}{dx}$

Ans: $\dfrac{dy}{dx}=\dfrac{1}{2}$, Take $\sqrt{1\pm\sin x}=\cos\dfrac{x}{2}\pm\sin\dfrac{x}{2}$

33) Let $y=\tan^{-1}\left[\dfrac{4x}{1+5x^2}\right]+\tan^{-1}\left[\dfrac{2+3x}{3-2x}\right]$, show that $\dfrac{dy}{dx}=\dfrac{5}{1+25x^2}$

34) Prove that $\dfrac{d}{dx}\left[\dfrac{1}{4\sqrt{2}}\log\left|\dfrac{x^2+\sqrt{2}x+1}{x^2-\sqrt{2}x+1}\right|+\dfrac{1}{2\sqrt{2}}\tan^{-1}\dfrac{\sqrt{2}x}{1-x^2}\right]=\dfrac{1}{1+x^4}$

35) If $(x-a)^2 + (y-b)^2 = c^2$ for some $c > 0$ prove that

$$\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$ is a constant independent of a and b. Ans: = -c

36) If $\sqrt{1-x^2}+\sqrt{1-y^2} = a(x-y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$

37) If $y = \frac{1}{\sqrt{b^2-a^2}}\log\left[\frac{\sqrt{b+a}+\sqrt{b-a}\tan\frac{x}{2}}{\sqrt{b+a}-\sqrt{b-a}\tan\frac{x}{2}}\right]$ prove that $\frac{dy}{dx} = \frac{\sec^2\frac{x}{2}}{(b+1)-(b-a)\tan^2\frac{x}{2}}$

38) If $y = |\cos x| + |\sin x|$, find $\frac{dy}{dx}$ at $x = \frac{2\Pi}{3}$

Ans: $\frac{1}{2}\left(\sqrt{3}-1\right)$; Take $y = -\cos x + \sin x$ around $x = \frac{2\Pi}{3}$

39) Find $\frac{dy}{dx}$ when $y = \tan^{-1}\left[\frac{x^{\frac{1}{3}}+a^{\frac{1}{3}}}{1-x^{\frac{1}{3}}a^{\frac{1}{3}}}\right]$ Ans: $\frac{1}{3x^{\frac{2}{3}}\left(1+x^{\frac{2}{3}}\right)}$

40) If $y = f\left(\frac{2x-1}{x^2+1}\right)$ and $f^{I}(x) = \sin x^2$, find $\frac{dy}{dx}$ Ans: $\frac{dy}{dx} = \frac{-2x^2+2x+2}{(x^2+1)^2}\operatorname{Sin}\left(\frac{2x-1}{x^2+1}\right)^2$

41) If $y^{\cos x} + (\tan^{-1}x)^y = 1$, find $\frac{dy}{dx}$

Ans: $\frac{dy}{dx} = \frac{y^{\cot x}.\operatorname{cosec}^2 x.\log y - \left\{\frac{(\tan^{-1}x)^{y-1}.y}{1+x^2}\right\}}{y^{\cot x-1}.\cot x + (\tan^{-1}x)^y.\log(\tan^{-1}x)}$ Use logarithmic differentiation.

42) If $x = \cos\theta + \log\tan\frac{\theta}{2}$, $y = \sin\theta$ find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\Pi}{4}$ Ans: $2\sqrt{2}$

43) Differentiate $\operatorname{Cos}^{-1}\left[\frac{3\cos x-2\sin x}{\sqrt{13}}\right]$ w.r.t. $\operatorname{Sin}^{-1}\left[\frac{5\sin x+4\cos x}{\sqrt{41}}\right]$ Ans: 1

44) If $y = \sqrt{x^2+1} - \log\left(\frac{1}{x}+\sqrt{1+\frac{1}{x^2}}\right)$, find $\frac{dy}{dx}$ Ans: $\frac{dy}{dx} = \frac{x^2+\sqrt{1+x^2}+1}{x(1+\sqrt{1+x^2}}$

45) If $x\sqrt{1+y}+y\sqrt{1+x}$, prove that $\frac{dy}{dx} = \frac{-1}{(1+x)^2}$

46) Verify Rolle's theorem for the function $f(x) = e^{1-x^2}$ in the interval [-1,1] Ans: C = 0

47) It is given that for the function $f(x) = x^3-6x^2+px+q$ on [1,3]. Rolle's theorem holds with

$C = 2+\frac{1}{\sqrt{3}}$. Find the values of p and q.

48) If f(x) and g(x) are functions derivable in [a,b] such that f(a) = 4, f(b) = 10, g(a) =1, g(b) =3. Show that for $a < c < b$, we have $f^{I}(c) = 3g^{I}(c)$.

49) Using LMV Theorem, find a point on the curve $y = (x-3)^2$, where the tangent is parallel to the chord joining (3,0) and (5,4) Ans: (4,1)

50) Verify the Rolle's Theorem for the function f(x) = Sinx-Cosx in the interval $\left[\frac{\Pi}{4},\frac{5\Pi}{4}\right]$

Ans: $C=\frac{3\Pi}{4}$

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APPLICATION OF DERIVATIVES

1) The slope of the tangent to the curve represented by $x = t^2+3t-8$ and $y = 2t^2-2t-5$ at the point M(2,-1) is

(a) $7/6$ (b) $2/3$ (c) $3/2$ (d) $6/7$ Ans (d)

2) The function $f(x) = 2 \log (x-2) - x^2+4x+1$ increases in the interval.

(a) (1,2) (b) (2,3) (c) $\left(5/2, 3\right)$ (d) (2,4) Ans: (b) and (c)

3) The function $y = \tan^{-1}x - x$ decreases in the interval of

(a) $(1, \infty)$ (b) $(-1, \infty)$ (c) $(-\infty, \infty)$ (d) $(0, \infty)$ Ans: all

4) The value of a for which the function $f(x) = a\sin x + \frac{1}{3}\sin 3x$ has an extreme at $x = \Pi/3$ is

(a) 1 (b) −1 (c) (O) (d) 2 Ans: d

5) The co-ordinates of the point p(x,y) in the first quadrant on the ellipse $\frac{x^2}{8}+\frac{y^2}{18}=1$ so that the area of the triangle formed by the tangent at P and the co-ordinate axes is the smallest are given by

(a) (2,3) (b) $\sqrt{8},0$ (d) $\left(\sqrt{18},0\right)$ (d) none of these Ans: (a)

6) The difference between the greatest and the least values of the function

$f(x) = \cos x + \frac{1}{2}\cos 2x - \frac{1}{3}\cos 3x$ is

(a) $2/3$ (b) $8/7$ (c) $9/4$ (d) $3/8$ Ans: (c)

7) If $y = a \log|x| + bx^2+x$ has its extreme values at x = −1 and x = 2 then

(a) a=2 b= −1 (b) a=2, b = −½ (c) a=−2, b=½ (d) none of these Ans: (b)

8) If θ is the semivertical angle of a cone of maximum volume and given slant height, then $\tan\theta$ is given by

(a) 2 (b) 1 (c) $\sqrt{2}$ (d) $\sqrt{3}$ Ans: (c)

9) If $f(x) = \frac{x}{\sin x}$ and $g(x) = \frac{x}{\tan x}$ where $0 \le x \le 1$ then in this interval

(a) both f(x) and g(x) are increasing
(b) both f(x) and g(x) are decreasing
(c) f(x) is an increasing function
(d) g(x) is an increasing function Ans: (c)

10) If $f(x) = \begin{cases} 3x^2+12x-1 & : -1 \le x \le 2 \\ 37-x & : 2 < x \le 3 \end{cases}$ then

(a) f(x) is increasing on [−1,2]
(b) f(x) is continuous on [−1,3]
(c) $f^{I}(2)$ doesn't exist
(d) f(x) has the maximum value at x = 2 Ans: (a),(b),(c),(d)

11) The function $\frac{\sin(x+\alpha)}{\sin(x+\beta)}$ has no maximum or minimum value if

(a) $\beta-\alpha = k\Pi$ (b) $\beta - \alpha \ne k\Pi$ (c) $\beta - \alpha = 2k\Pi$
(d) None of these where K is an integer. Ans: (b)

12) If $f(x)=\frac{x^2-1}{x^2+1}$ for every real number then minimum value of f

(a) does not exist (b) is not attained even though f is bounded
(c) is equal to 1 (d) is equal to −1 Ans: (d)

13) If the line ax+by+c = 0 is normal to the curve xy = 1 then

(a) a>0, b>0 (b) a>0, b<0 (d) a<0, b>0 (d) a<0, b<0 Ans: (b),(c)

14) The tangent to the curve
$x=a\sqrt{\cos 2\theta}\cos\theta \quad y=a\sqrt{\cos 2\theta}\sin\theta$ at the point corresponding to $\theta=\Pi/6$ is

(a) Parallel to the x-axis (b) Parallel to the y-axis
(c) Parallel to the line y = x (d) none of these Ans: (a)

15) The minimum value of f(x) = |3−x| + |2+x| + |5−x| is

(a) 0 (b) 7 (c) 8 (d) 10 Ans: (b)

16) If $x=a(\theta-\sin\theta),\ y=a(1-\cos\theta)$ then $\frac{d^2y}{dx^2}$ at $\theta=\Pi/2$ is

(a) $\frac{1}{a}$ (b) $\frac{1}{2}$ (c) $-\frac{1}{a}$ (d) $-\frac{1}{2a}$ Ans: (c)

17) If $y=\log\tan\left(\frac{\Pi}{4}+\frac{\Pi}{2}\right)$ then $\frac{dy}{dx}$ is

(a) 0 (b) cosx (c) −secx (d) secx Ans: (d)

18) 'C' on LMV for f(x) = x^2−3x in [0,1] is

(a) 0 (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) does not exist Ans: (b)

19) The values of a and b for which the function $f(x)=\begin{cases}ax+1 & x\le 3\\ bx+3 & x>3\end{cases}$ is continuous at x = 3 are

(a) 3a + 2b = 5 (b) 3a = 2+3b (c) 3,2 (d) none of these Ans: (b)

20) The tangents to the curve y = x^3+6 at the points (-1,5) and (1,7) are

(a) Perpendicular (b) parallel (c) coincident (d) none of these Ans: (b)

21) If $\frac{dy}{dx}=0$ then the tangent is

(a) Parallel to x-axis (b) parallel to y-axis (c) Perpendicular to x-axis
(d) perpendicular to y-axis Ans: (a)

22) If the slope of the tangent is zero at (x_1, y_1) then the equation of the tangent at (x_1,y_1) is

(a) y_1=mx_1+c (b) y_1= mx_1 (c) y−y_1 (d) y=0 Ans: (c)

23) The function $f(x)=-\frac{x}{2}+\sin x$ is always increasing in

(a) $\left(\frac{-\Pi}{2},\frac{\Pi}{2}\right)$ (b) $\left(0,\frac{\Pi}{4}\right)$ (c) $\left(\frac{\Pi}{4},\frac{\Pi}{2}\right)$ (d) $\left(\frac{-\Pi}{3},\frac{\Pi}{3}\right)$ Ans: (d)\

24) The least value of 'a' such that the function f(x) = x^2+ax+1 is strictly increasing on (1,2) is

(a) −2 (b) 2 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$ Ans: (a)

25) The least value of f(x) = tan-1 (sinx + cosx) strictly increasing is

(a) $\left(\frac{\Pi}{4},\frac{\Pi}{2}\right)$ (b) $\left(0,\frac{\Pi}{2}\right)$ (c) 0 (d) none of these Ans: (d)

4-6 Marks

26) Determine the points of maxima and minima of the function

$f(x)=\frac{1}{8}\log x-bx+cx^2$ where $b\geq 0$

Ans: f has maxima at $2=\frac{1}{4}\left(b-\sqrt{b^2-1}\right)$ and minima at $\beta=\frac{1}{4}\left(b+\sqrt{b^2-1}\right)$

27) Find the interval in which the following functions are increasing or decreasing

(a) $y=\log\left(x+\sqrt{1+x^2}\right)$ (b) $y=\frac{10}{4x^3-9x^2+6x}$

Ans: (a) increases on $(-\infty,\infty)$ (b) increases on $\left(\frac{1}{2},1\right)$ and decreases on $(-\infty,0)\cup\left(0,\frac{1}{2}\right)\cup(1,\infty)$

28) Find the equation of normal to the curve $y=(1+x)^y+\sin^{-1}(\sin^2 x)$ at $x=0$ Ans: x+y=1

29) If P_1 and P_2 are the lengths of the perpendiculars from origin on the tangent and normal to the curve $x^{2/3}+y^{2/3}=a^{2/3}$ respectively Prove that $4P_1^2+P_2^2=a^2$

30) What angle is formed by the y-axis and the tangent to the parabola $y = x^2+4x-17$ at the point $P\left(5/2,-3/4\right)$? Ans: $\theta=\Pi/2-\tan^{-1}9$

31) A cone is circumscribed about a sphere of radius r. Show that the volume of the cone is maximum when its semi vertical angle is $\text{Sin}^{-1}\left(\frac{1}{3}\right)$

32) Find the interval in which the function f(x) is increasing or decreasing

$f(x)=x^3-12x^2+36x+17$ Ans: Increasing in x<2 or x>6
Decreasing 2<x<6

33) Find the equation of tangent to the curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$ at the point (x_1,y_1) and show that the sum is intercepts on the axes is constant.

34) Find the point on the curve $y = x^3-11x+5$ at which the tangent has equation $y = x-11$
Ans: (2,—9)

35) Find the equation of all lines having slope —1 and that are tangents to the curve

$y=\frac{1}{x-1}, x\neq 1$ Ans: x+y+1=0 ; x+y—3=0

36) Prove that the curves $y^2 = 4ax$ and $xy = c^2$ cut at right angles if $c^4 = 32a^4$

37) Find the points on the curve $y = x^3 - 3x^2+2x$ at which the tangent to the curve is parallel to the line $y - 2x + 3 = 0$. Ans: (0,0) (2,0)

38) Show that the semi vertical angle of a right circular cone of maximum volume and given slant height is $\tan^{-1}\sqrt{2}$

39) Show that the semi vertical angle of a right circular cone of given total surface area and maximum volume is $\sin^{-1}\frac{1}{3}$

40) Show that the right circular cone of least curved surface area and given volume is $\sqrt{2}$ times the radius of the base.

41) Show that the height of a cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$

42) Find the absolute maximum and absolute minimum value of $f(x) = 2\cos x + x \quad x \varepsilon [\,0\,,\Pi\,]$

Ans: max at $x = \Pi/6$ and min at $x = 5\Pi/6$

43) Show that the volume of greatest cylinder which can be inscribed in a cone of height h and semi vertical angle α is $\frac{4}{27}\Pi h^3 \tan^2\alpha$

44) A window is in the form of a rectangle above which there is a semicircle. If the perimeter of the window is 'P' cm. Show that the window will allow the maximum possible light only when the radius of the semi circle is $\frac{P}{\Pi + 4}$ cm

45) Find the area of the greatest isoscles triangle that can be inscribed in a given ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with its vertex coinciding with one extremity of the major axis.

46) A rectangular window is surmounted by an equilateral triangle. Given that the perimeter is 16 cm. Find the width of the window so that the maximum amount of light may enter.

47) Show that the volume of the greatest cylinder which can be inscribed in a cone of height h and semi vertical angle 30° is $\frac{4}{81}\Pi h^3$

48) Show that the height of the right circular cylinder of maximum volume that can be inscribed in a given right circular cone of height h is $h/3$.

49) Of all the rectangles each of which has perimeter 40 metres find one which has maximum area. Find the area also.

50) Show that the rectangle of maximum area that can be inscribed in a circle of radius 'r' cms is a square of side $\sqrt{2}\, r$

51) Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

52) Show that the semi vertical angle of the cone of maximum volume and of given slant height is $\tan^{-1}\sqrt{2}$

53) Given the sum of the perimeters of a square and a circle show that the sum of their areas is least when the side of the square is equal to the diameter of a circle.

54) Find the maximum slope of the curve $f(x) = 2x + 3x^2 - x^3 - 27$

55) Show that the height of the cone of maximum volume that can be inscribed in a sphere of radius 12 cm is 16 cm.

56) A point on the hypotenuse of a right angled triangle is at distances a and b from the sides. Show that the length of the hypotenuse is at least $\left(a^{2/3} + b^{2/3}\right)^{3/2}$

57) Sand is pouring from a pipe at the rate of $12cm^3$/sec. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone in increasing when the height is 4 cm?

Ans : $\frac{1}{48\Pi}$ cm/sec.

58) A man 160 cm tall walks away from a source of light situated at the top of the pole 6m high at the rate of 1.1 m/sec. How fast is the length of the shadow increasing when he is 1m away from the pole. Ans: 0.4 cm/sec.

59) An edge of a variable cube is increasing at the rate of 5 cm/sec. How fast is the volume of the cube is increasing when edge is 10cm long? Ans: 1500 cm^3/sec.

60) A balloon which always remains spherical is being inflated by pumping in gas at the rate of 900 cm^3/sec. Find the rate at which the radius of the balloon is increasing when the radius of the balloon is 15 cm. Ans: $\frac{1}{\Pi}$ cm/sec.

61) The volume of a spherical balloon is increasing at the rate of 25 cm^3/sec. Find the rate of change of its surface area at the instant when the radius is 5 cm. Ans: 10 cm^2/sec.

62) The surface area of a spherical bubble is increasing at the rate of 2 cm^2/sec. Find the rate at which the volume of the bubble is increasing at the instant if its radius is 6 cm.

Ans: $80\Pi cm^2$ / sec.

63) Gas is escaping from a spherical balloon at the rate of 900 cm^3/sec. How fast is the surface area, radius of the balloon shrinking when the radius of the balloon is 30cm?

Ans: $\frac{dA}{dt} = 60cm^2$ / sec. $\frac{dr}{dt} = \frac{1}{4\Pi}$ cm/sec.

64) Water is passed into an inverted cone of base radius 5 cm and depth 10 cm at the rate of $\frac{3}{2}$ c.c./sec. Find the rate at which the level of water is rising when depth is 4 cm.

Ans: $\frac{3}{8\Pi}$ cm/sec.

65) Show that the function $f(x) = e^{2x}$ is strictly increasing on R.

66) Show that $f(x) = 3x^5 + 40x^3 + 240x$ is always increasing on R.

67) Find the interval in which the function $f(x) = x^4 - 4x^3 + 4x^2 + 15$ is increasing or decreasing.

68) Find whether $f(x) = \cos(2x + \Pi/4)\frac{3\Pi}{8} < x < \frac{5\Pi}{8}$ is increasing or decreasing.

69) Find the interval in which the function $\frac{4.\sin x - 2x - x\cos x}{2 + \cos x}$ is increasing or decreasing.

Ans: $(0, \Pi/2) \cup \left(\frac{3\Pi}{2}, 2\Pi\right)$

70) FInd the interval in which $f(x) = 8 + 36x + 3x^2 - 2x^3$ is increasing or decreasing.

Ans: Increasing : $2 < x < 3$ decreasing : $x > 3$ or $x < -2$

71) Find the interval on which the function $\frac{x}{\log x}$ is increasing or decreasing.

Ans: Increasing in ((e, ∞)

decreasing for (0,1)U(1,e)

72) Find the points on the curve $\frac{x^2}{4}+\frac{y^2}{25}=1$ at which the tangents are parallel to x-axis and parallel to y-axis. Ans: $(0,\pm5), (\pm2,0)$

73) Find equation of tangents to the curve $x = a\cos^3\theta,\ y = b\sin^3\theta$ at $\theta = \Pi/4$

74) Find the equations of the normal lines to the curve $y = 4x^3-3x+5$ which are parallel to the line $9y+x+3 = 0$. Ans: $x+9y-55=0,\ x+9y-35=0$

75) FInd the equation of tangent and normal to the curve $y^2 = \frac{x^3}{4-x}$ at $(2, -2)$

Ans: $2x+y-2=0,\ x-2y-6=0$

76) FInd the equation of the tangent to the curve $\sqrt{x}+\sqrt{y}=a$ at the point $\left(\frac{a^2}{4},\frac{a^2}{4}\right)$

Ans: $2x+2y=a^2$

77) Find the angle between the parabolas $y^2=4ax$ and $x^2=4by$ at their point of intersection other than the origin. Ans: $\theta = \tan^{-1}\left[\frac{3a^{1/3}b^{1/3}}{2\left(a^{2/3}+b^{2/3}\right)}\right]$

78) Using differentials find the appropriate value of $(82)^{1/4}$ Ans: 3.0092

79) If $y = x^4-10$ and if x changes from 2 to 1.97, what is the appropriate change in y ?

Ans: -0.96, y changes from 6 to 5.04

80) Find the appropriate change in volume of a cube when side increases by 1%. Ans: 3%

81) Use differentials to evaluate $\left(\frac{17}{81}\right)^{1/4}$ approximately. Ans: 0.677

82) Using differentials evaluate tan 44° approximately, 1° = 0.07145°. Ans: 0.9651

83) Find the approximate value of x if $2x^4-160 = 0$ Ans: 2.991

84) Find the maximum and minimum values of 'f', if any of the function $f(x) = |x|,\ x \varepsilon R$

85) Find the maximum and minimum value of $f(x) = |(\sin 4x+5)|$ without using derivatives.

86) The curve $y = ax^3+6x^2+bx+5$ touches the x-axis at P(-2,0) and cuts the y-axis at a point Q where its gradient is 3. Find a,b,c.

Ans: $a = -\frac{1}{2}\ b = \frac{-3}{4},\ c = 3$

87) FInd the local maxima and local minima if any of the function $f(x) = e^{5x}$

88) Find the maxima or minima if any of the function $f(x) = \frac{1}{x^2+2}$

Ans: local max at x = 0, value $\frac{1}{2}$

89) Without using derivatives find the maximum or minimum value of $f(x) = -|x+5| + 3$

Ans: max value 3, no minimum value

90) Without using derivatives find the maximum and minimum value of f(x) = sin2x + 7

Ans: Max. value 8, min. value 6

91) FInd whether $f(x) = e^x$ has maxima or minima. Ans: No maxima nor minima

92) At what point in the interval $[0, 2\Pi]$ does the function Sin2x attain its maximum value?

93) Find the intervals in which f(x) = logcosx is strictly decreasing and strictly increasing.

Ans: decreasing $\left(0, \Pi/2\right)$, increasing $\left(\Pi/2, \Pi\right)$

94) Find the interval in which $y = x^2e^{-x}$ is increasing. Ans: (0,2)

95) Find two positive numbers x and y such that their sum is 16 and sum of whose cubes is minimum.

Ans: 8,8

96) Find the local maximum and local minimum value of the function.

$f(x) = \sin x + \frac{1}{2}\cos 2x$ in $\left[0, \Pi/2\right]$ Ans: local max. value $\frac{3}{4}$ at $x = \frac{\Pi}{6}$

97) Two equal sides of an isosceles triangle with fixed base b are decreasing at the rate of 3cm/sec. How fast is the area decreasing when the two equal sides are equal to the base?

98) A poster is to contain $50cm^2$ of matter with borders of 4 cm at top and bottom and of 2 cm on each side. Find the dimensions if the total area of the poster is minimum.

99) Find the sides of a rectangle of greatest area that can be inscribed in the ellipse $x^2 + 4y^2 = 16$

Ans: $4\sqrt{2}, 2\sqrt{2}$

100) Find the maximum profit that a company can make if the profit function is given by

$P(x) = 41 - 24x - 18x^2$ Ans: 49

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INTEGRATION

Objective Questions - choose the correct alternative

1) $\int \frac{\cos\sqrt{x}}{\sqrt{x}}dx =$

(a) $\sin\sqrt{x}$ (b) $2\cos\sqrt{x}$ (c) $2\sin\sqrt{x}$ (d) $\frac{\sqrt{\cos x}}{x}$

2) $\int \frac{\tan(\log x)}{x}dx =$

(a) log cos(logx) (b) log sec(logx) (c) log sin(logx) (d) −logcos(logx)

3) $\int \frac{\sqrt{\tan x}}{\sin x \cos x}dx =$

(a) $2\sqrt{\tan x}$ (b) $2\sqrt{\cot x}$ (c) $\sqrt{\cot x}$ (d) $\sqrt{\tan x}$

4) $\int \frac{dx}{\sin x + \sqrt{3}\cos x} =$

(a) $\frac{1}{2}\log\tan\left[\frac{x}{2}+\frac{\Pi}{6}\right]$ (b) $\frac{1}{2}\log\left\{\operatorname{cosec}\left(x+\frac{\Pi}{3}\right)-\cot\left(x+\frac{\Pi}{3}\right)\right\}$

(c) $\frac{1}{2}\log\left\{\sec\left(x-\frac{\Pi}{6}\right)+\tan\left(x-\frac{\Pi}{6}\right)\right\}$ (d) $-\frac{1}{2}\log\left\{\operatorname{cosec}\left(x+\frac{\Pi}{3}\right)+\cot\left(x+\frac{\Pi}{3}\right)\right\}$

5) $\int \frac{\sin 2x dx}{a^2\cos^2 x + b^2\sin^2 x} =$

(a) $(b-a)\log(a^2\cos^2 x + b^2\sin^2 x)$ (b) $\frac{1}{b-a}\log(a^2\cos^2 x + b^2\sin^2 x)$

(c) $\frac{1}{b^2-a^2}\log(a^2\cos^2 x + b^2\sin^2 x)$ (d) $\frac{1}{a^2+b^2}\log(a^2\cos^2 x + b^2\sin^2 x)$

6) $\int \frac{e^x dx}{e^{2x}+1} =$

(a) $\log(e^x + e^{-x})$ (b) $\log(e^{2x}+1)$ (c) $\tan^{-1}(e^x)$ (d) $\tan^{-1}(2e^x)$

7) $\int \frac{\sqrt{x}}{\sqrt{a^3-x^3}}dx =$

(a) $\frac{2}{3}\left(\frac{x}{a}\right)^{3/2}$ (b) $\frac{2}{3}\sin^{-1}\left(\frac{x}{a}\right)^{3/2}$ (c) $\frac{2}{3}\cos^{-1}\left(\frac{x}{a}\right)^{3/2}$ (d) $\frac{2}{3}\sin^{-1}\left(\frac{a}{x}\right)^{3/2}$

8) $\int x^3 e^{x^2} dx =$

(a) $x^2(e^{x^2}-1)$ (b) $\frac{1}{2}x^2(e^{x^2}-1)$ (c) $\frac{1}{2}e^{x^2}(x^2-1)$ (d) $\frac{1}{2}(e^{x^2}-1)$

9) $\int \frac{x\sin^{-1}x}{\sqrt{1-x^2}}dx =$

(a) $\sqrt{1-x^2}\sin^{-1}x$ (b) $x\sin^{-1}x$ (c) $x - \sqrt{1-x^2}\sin^{-1}x$ (d) $(\sin^{-1}x)^2$

10) $\int x\tan^{-1}x\,dx =$

(a) $\left(\frac{x^2+1}{2}\right)\tan^{-1}x - \frac{x}{2}$ (b) $\left(\frac{x^2+1}{2}\right) + \tan^{-1}x - x$ (c) $(x^2+1)\tan^{-1}x - x$ (d) $(x^2+1)\tan^{-1}x + x$

11) $\int \frac{x+\sin x}{1+\cos x}dx =$

(a) $\tan\frac{x}{2}$ (b) $x\tan\frac{x}{2}$ (C) $\cot\frac{x}{2}$ (d) $x\cot\frac{x}{2}$

12) $\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x.\cos^2 x}dx =$

(a) $\tan x + \cot x$ (b) $\tan x - \cot x$ (c) $\tan x + \sec x$ (d) $\tan x + \operatorname{cosec} x$

13) $\int \frac{dx}{x\sqrt{1-x^3}} =$

(a) $\frac{1}{3}\log\left|\frac{\sqrt{1-x^2}+1}{\sqrt{1-x^2}-1}\right|$ (b) $\frac{1}{3}\log\left|\frac{\sqrt{1-x^2}-1}{\sqrt{1-x^3}+1}\right|$

(c) $\frac{2}{3}\log\left|\frac{1}{\sqrt{1-x^3}}\right|$ (d) $\frac{1}{3}\log\left|1-x^3\right|$

14) $\int \frac{dx}{x-X^3} =$

(a) $\frac{1}{2}\log\frac{x^2}{1-x^2}$ (b) $\frac{1}{2}\log\frac{1-x^2}{x^2}$ (c) $\log\frac{(1-x)}{x(1+x)}$ (d) $\log x(1-x^2)$

15) $\int \sqrt{1+x^2}\,d(x^2) =$

(a) $\frac{2}{3x}(1+x^2)^{3/2}$ (b) $\frac{2}{3}(1+x^2)^{3/2}$ (c) $\frac{2x}{3}(1+x^2)^{3/2}$ (d) $\frac{2x}{3}(1+x^2)$

16) $\int \frac{2x\,dx}{1+x^4} =$

(a) $\tan^{-1}(x^2)$ (b) $\frac{1}{2}\tan^{-1}x^2$ (c) $\log(1+x^4)$ (d) $\tan^{-1}\left(\frac{1}{x^2}\right)$

17) $\frac{3^x dx}{\sqrt{1-9^x}} =$

(a) $\log_e 3 \sin^{-1}(3^x)$ (b) $\frac{1}{\log_e 3}\sin^{-1}(3^x)$ (c) $\log_e 3 \sin^{-1}(3^{x/2})$ (d) $\frac{1}{\log_e 3}\sin^{-1}(3^{x/2})$

18) $\frac{1}{x(x^4-1)}dx =$

(a) $\log\left[\frac{x^4}{x^4-1}\right]$ (b) $\frac{1}{2}\log\left[\frac{x^2+1}{x^2+1}\right]$ (c) $\frac{1}{4}\log\left[\frac{x^4-1}{x^4}\right]$ (d) $\log\frac{x(x^2-1)}{x^2+1}$

19) $\int \frac{e^{\log}\left(1+\frac{1}{x^2}\right)dx}{x^2+\frac{1}{x^2}} =$

(a) $\frac{1}{\sqrt{2}}\tan^{-1}\left(x+\frac{1}{x}\right)$ (b) $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x-\frac{1}{x}}{\sqrt{2}}\right)$ (c) $\sqrt{2}\tan^{-1}\left(x+\frac{1}{x}\right)$ (d) $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{1}{x}-x\right)$

20) $\int \frac{dx}{(2+x)\sqrt{1+x}} =$

(a) $2\tan^{-1}\sqrt{1+x}$ (b) $\frac{1}{2}\tan^{-1}\sqrt{1+x}$ (c) $\tan^{-1}\sqrt{1+x}$ (d) $\log\{(2+x)\sqrt{1+x}\}$

21) $\int \frac{dx}{x+\sqrt{x}} =$ (a) $\log(1+\sqrt{x})$ (b) $\frac{1}{2}\log(1+\sqrt{x})$ (c) $2\log(1+\sqrt{x})$ (d) $\log(x+\sqrt{x})$

22) Ssin2x log cosx dx =

(a) $\frac{1}{2}\cos^2 x - \cos^2 x\log\cos x$ (b) $\frac{1}{2}\cos^2 x + \cos^2\log\cos x$ (c) $\cos^2 x.\log\cos x$ (d) $\cos^2 x(1-\log\cos x)$

23) If $\int \frac{dx}{(x+1)(x-2)} = A\log(x+1) + B\log(x-2) + C$, then

(a) A+B = 0 (b) AB = — (c) AB = $\frac{1}{9}$ (d) AB = —9

24) If $\int \tan^4 x\, dx = a\tan^3 x + b\tan x + cx$, then

(a) $a = \frac{1}{3}$ (b) b = —1 (c) a = 1 (d) c = 1

25) If $\int \frac{\cos x}{\cos(x-a)}dx = Ax + B\log\cos(x-\alpha)$, then

(a) $A = \cos\alpha$ (b) $B = \sin\alpha$ (c) $A = \sin\alpha$ (d) $B = \cos\alpha$

Answers :

1. (c) 2. (b), (d) 3. (a) 4. (a), (b), (c), (d) 5. (c) 6. (c) 7. (b) 8. (c) 9. (c) 10. (a) 11. (b) 12. (a) 13. (b) 14. (a) 15. (b) 16. (a) 17. (b) 18. (c) 19. (b) 20. (a) 21. (c) 22. (a) 23. (a), (c) 24. (a), (b), (d) 25. (a), (b).

EVALUATE

26) $\int |x|\,dx$ Ans: $\frac{x}{2}|x|+c$

27) $\int\left(10^{x}+10x+\frac{10}{x}+\frac{x}{10}+x^{10}+10^{10}\right)dx$

Ans: $\frac{10^{x}}{\log 10}+\frac{101}{20}x^{2}+10\log|x|+\frac{x^{11}}{11}+10^{10}x+c$

28) $\int\frac{\left(a^{x}+b^{x}\right)^{2}}{a^{x}b^{x}}dx$ Ans: $\frac{\left(\frac{a}{b}\right)^{x}}{\log a/b}+\frac{\left(\frac{b}{a}\right)^{x}}{\log b/a}+2x+c$

29) $\int\frac{1}{1+\sin x}dx$ Ans: $\tan x - \sec x + c$

30) $\int\cos^{-1}(\sin x)dx$ Ans: $\frac{\Pi}{2}x-\frac{x^{2}}{2}+c$

31) $\int 3^{-2x}e^{-2x}dx$ Ans: $\frac{(3e)^{-2x}}{-2\log 3e}$

32) $\int\frac{1}{\sin^{2}x\cos^{2}x}dx$ Ans: $\tan x - \cot x + c$

33) $\int\frac{e^{2\log x}-1}{e^{2\log x}+1}dx$ Ans: $x - 2\tan^{-1}x + c$

34) $\int \mathrm{Sin}^{2}\left(2\tan^{-1}\sqrt{\frac{1+x}{1-x}}\right)dx \quad -|\leq x\leq 1$ Ans: $x-\frac{x^{3}}{3}+c$

35) $\int\frac{\sin x+\cos x}{\sqrt{1+\sin 2x}}dx$ Ans: $x+c$

36) $\int\frac{\sin 2x}{a^{2}\cos^{2}x+b^{2}\sin^{2}x}dx$ Ans: $\frac{1}{b^{2}-a^{2}}\log(a^{2}\cos^{2}x+b^{2}\sin^{2}x)+c$

37) $\int\frac{1+\sin 2x}{x+\sin^{2}x}dx$ Ans: $\log|x+\sin^{2}x|+c$

38) $\int\frac{\sqrt{\tan x}}{\sin x\cos x}dx$ Ans: $2\sqrt{\tan x}+c$

39) $\int\sqrt{\frac{a-x}{a+x}}dx$ Ans: $a\sin^{-1}\frac{x}{a}+\sqrt{a^{2}-x^{2}}+c$

40) $\int\frac{1}{e^{x}-1}dx$ Ans: $\log|e^{x}-1|-x+c$

41) $\int\frac{1+\tan x}{x+\log\sec x}dx$ Ans: $\log|x+\log\sec x|+c$

42) $\int \frac{1}{1+\sqrt{x}}dx$ Ans : $2\log(\sqrt{x}+1)+c$

43) $\int x^2\sqrt[3]{2x-1}\,dx$ Ans : $\frac{3}{1120}(2x-1)^{4/3}(56x^2+24x+9)+c$

44) $\frac{dx}{\sqrt{\sin^3 x\sin(x+\alpha)}}$ Hint : Put $\sqrt{\frac{\sin(x+\alpha)}{\sin x}}=t$ Ans: $\frac{-2}{\sin\alpha}\sqrt{\frac{\sin(x+\alpha)}{\sin x}}+c$

45) $\int e^{\sin^2 x}\sin 2x\,dx$ Ans : $e^{\sin 2x}+c$

46) $\int \frac{dx}{a^2\sin^2 x+b^2\cos^2 x}$ Ans: $\frac{1}{ab}\tan^{-1}\left(\frac{a\tan x}{b}\right)+c$

47) $\int \frac{dx}{(x+1)\sqrt{x^2+1}}$, $x>-1$ Hint : $-x+1=\frac{1}{t}$ Ans : $-\frac{1}{\sqrt{2}}\log\left\{\frac{1-x\pm\sqrt{2+2x^2}}{x+1}\right\}+c^l$

48) $\int \frac{1}{4+5\cos x}dx$ Ans: $\frac{1}{3}\log\left|\frac{3+\tan x/2}{3-\tan x/2}\right|+c$

49) $\int \frac{2\sin x+3\cos x}{3\sin x+4\cos x}dx$ Ans : $\frac{18}{25}x+\frac{1}{25}\log|3\sin x+4\cos x|+c$

50) $\int \frac{\sin^{-1}\sqrt{x}-\cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x}+\cos^{-1}\sqrt{x}}dx$ Ans : $\frac{2}{\Pi}\left\{\sqrt{x-x^2}-(1-2x)\sin^{-1}\sqrt{x}\right\}-x+c$

51) $\int \frac{1}{x^4+1}dx$ Ans: $\frac{1}{2\sqrt{2}}\tan^{-1}\frac{x^2-1}{\sqrt{2}x}-\frac{1}{4\sqrt{2}}\log\left|\frac{x^2-\sqrt{2}x+1}{x^2+\sqrt{2}x+1}\right|+c$

52) $S\sqrt{\tan\theta}\,d\theta$ Ans: $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan\theta-1}{\sqrt{2}\tan\theta}\right)+\frac{1}{2\sqrt{2}}\log\left|\frac{\tan\theta-\sqrt{2}\tan\theta+1}{\tan\theta+\sqrt{2}\tan\theta}\right|+c$

53) $\int 2^{2^{2^x}}2^{2^x}2^x dx$ Ans: $\frac{1}{(\log 2)^3}2^{2^{2^x}}+c$

54) $\int \frac{\sin x+\cos x}{\sin^4 x+\cos^4 x}dx$ Ans: $\frac{1}{\sqrt{2}}\left\{\frac{1}{2\sqrt{2}+1}\log\left|\frac{\sqrt{\sqrt{2}+1}+t}{\sqrt{\sqrt{2}+1}-t}\right|+\frac{1}{2\sqrt{\sqrt{2}-1}}\tan^{-1}\frac{t}{\sqrt{\sqrt{2}-1}}\right\}+c$

Where t = sinx — cosx

55) $\int \frac{8}{(x+2)(x^2+4)}dx$ Ans: $\log(x+2)-\frac{1}{2}\log(x^2+4)+\tan^{-1}\frac{x}{2}+c$

56) $\int \frac{\cos x}{(1+\sin x)(2+\sin x)}dx$ Ans: $\log\left(\frac{1+\sin x}{2+\sin x}\right)+c$

57) $\int \frac{\sqrt{\cos 2x}}{\sin x}dx$ Ans: $\frac{1}{2}\log\left|\frac{1+t}{1-t}\right|+\frac{1}{\sqrt{2}}\log\left|\frac{\sqrt{2}+t}{\sqrt{2}-t}\right|+c$ Where t = $\sqrt{1-\tan^2 x}$

58) $\int \frac{dx}{(\sin x - 2\cos x)(2\sin x + \cos x)}$ Ans: $\frac{1}{5}\log\left|\frac{\tan x - 2}{2\tan x + 1}\right| + c$

Hint: divided n r and D r by $\cos^2 x$

59) $\int \sqrt{\frac{x}{a^3 - x^3}}\,dx$ Ans : $\frac{2}{3}\sin^{-1}\frac{x^{3/2}}{a^{3/2}} + c$

60) $\int \frac{dx}{x(x^7 + 1)}$ Ans: $\frac{1}{7}\log\left|\frac{x^7}{x^7 + 1}\right| + c$

61) $\int_{\Pi/3}^{\Pi/2} \frac{\sqrt{1 + \cos x}}{(1 - \cos x)^{7/2}}\,dx$ Ans : $\frac{7}{3}$

62) $\int_0^{\Pi/4} \log(1 + \tan\theta)\,d\theta$ Ans: $\frac{\Pi}{8}\log 2$

63) $\int_0^{\Pi/2} \frac{dx}{(a^2\sin^2 x + b^2\cos^2 x)^2}$ Ans: $\frac{\Pi(a^2 + b^2)}{4a^3b^3}$ Hint: divide Nr and Dr by $\cos^4 x$

64) $\int_0^{\Pi/3} \frac{\cos x}{3 + 4\sin x}\,dx$ Ans: $\frac{1}{4}\log\left(\frac{3 + 2\sqrt{3}}{3}\right)$

65) $\int_0^1 \frac{x}{1 + x^4}\,dx$ Ans: $\frac{\Pi}{8}$

66) $\int_0^{3/2} |x\cos \Pi x|\,dx$ Ans: $\frac{5\Pi - 2}{2\Pi^2}$

67) $\int_0^{\Pi/2} \frac{1}{2\cos x + 4\sin x}\,dx$ Ans: $\frac{1}{\sqrt{5}}\log\frac{3 + \sqrt{5}}{2}$

68) $\int_0^2 |x^2 + 2x - 3|\,dx$ Ans : 4

69) Evaluate $\int_{-\Pi/2}^{\Pi/2} \cos^4 x\,dx$ Ans: $\frac{3\Pi}{8}$

70) $\int_0^{\Pi} \frac{x\sin x}{1 + \cos^2 x}\,dx$ Ans : $\frac{\Pi^2}{4}$

71) $\int_0^{\Pi} \log(1 + \cos x)\,dx$ ans : $-\Pi\log 2$

72) $\int_0^{\Pi} \frac{x\tan x}{\sec x \operatorname{cosec} x}$ Ans : $\Pi/2$

73) $\int_0^1 x(1-x)^n dx$ Ans : $\frac{1}{n+1} - \frac{1}{n+2}$

74) $\int_0^{\Pi/2} \log \tan x$ Ans : $I = 0$

75) $\int^0 \frac{xdx}{1+\sin x+\cos x}$ Ans : $\Pi/2 \log 2$

76) $\int_{-\Pi/2}^{\Pi/2} \cos^4 x\, dx$ Ans : $\frac{3\Pi}{8}$

77) $\int_{-5}^{8} |x+3| dx$

78) $\int_0^{2\Pi} \frac{\sin 2\theta}{a - b\cos\theta} d\theta$ $a > b > 0$ Ans: $I = 0$

79) $\int_0^1 \frac{dx}{\sqrt{x+1}+\sqrt{x}}$ Ans : $\frac{2}{3}\left(3^{3/2} - 1\right)$

80) $\int_0^{\Pi/4} \sin 2x \sin 3x\, dx$ Ans : $\frac{3}{5}$

81) $\int_0^{\Pi} \frac{x\, dx}{a^2\cos^2 x + b^2 \sin^2 x}$ Ans : $\frac{\Pi^2}{2ab}$

82) $\int_{-\Pi/4}^{\Pi/4} |\sin x|\, dx$ Ans : $2 - \sqrt{2}$

83) $\int_{-\Pi/4}^{\Pi/4} |\sin x|\, dx$ Ans : $2 - \sqrt{2}$

84) $\int_0^{\Pi/2} \frac{\sin x - \cos x}{1+\sin x \cos x} dx$ Ans : 0

85) $\int_0^a \frac{dx}{x+\sqrt{a^2-x^2}}$ Ans : $\Pi/4$

86) $\int_{1}^{4}[|x-1|+|x-2|+|x-3|]dx$ Ans : $\frac{19}{2}$

87) $\int_{-\Pi}^{\Pi} x^{20}\sin^{9}x\,d\,x$ Ans : 0

88) $\int_{3}^{9}\frac{\sqrt{12-x}}{\sqrt{x}+\sqrt{12-x}}dx$ Ans : 3

89) $\int_{-1}^{1}\log\left(\frac{4-x}{4+x}\right)dx$ Ans : 0

90) $\int_{0}^{\Pi/2}\frac{1}{2\cos x+4\sin x}dx$ Ans : $\frac{1}{\sqrt{5}}\log\frac{3+\sqrt{5}}{2}$

91) $\int_{0}^{\Pi/2}\log(\cos\theta)d\theta$ Ans : $\frac{-\Pi}{2}\log 2$

92) $\int_{-\Pi}^{\Pi}\frac{2x(1+\sin x)}{1+\cos^{2}x}$ Ans : Π^{2}

93) $\int_{0}^{1}x\sqrt{\frac{1-x^{2}}{1+x^{2}}}dx$ Ans : $\frac{\Pi}{4}-\frac{1}{2}$

94) $\int_{0}^{\Pi/2}\frac{1}{\cos\left(x-\Pi/3\right)\cos\left(x-\Pi/6\right)}$ Ans : $2\log\sqrt{3}+\log 2$

95) $\int_{-\Pi}^{\Pi}\left(\sin^{-93}x+x^{295}\right)$ Ans : 0

96) $\int_{0}^{16}\frac{x^{1/4}}{1+x^{1/2}}dx$ Ans : $\Pi/4$

97) $\int_{0}^{\Pi/2}\frac{\sqrt{\tan x}}{1+\sqrt{\tan x}}dx$ Ans : $\Pi/4$

98) $\int_{0}^{1}\sqrt{\frac{1-x}{1+x}}dx$ Ans : $\frac{\Pi}{2}-1$

99) $\int_{0}^{\Pi/4}2\tan^{3}x\,dx$ Ans : 1 - log2

100) $\int_{\Pi/6}^{\Pi/3}\frac{dx}{1+\sqrt{\tan x}}$ Ans : $\Pi/12$

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APPLICATION OF INTEGRALS

1) Find the area bounded by the curve y = 2 cosx and the x-axis from x = 0 to x = $x = 2\Pi$

Ans: 8 sq.units.

2) Find the area bounded by the x-axis part of the curve $y = 1 + \frac{8}{x^2}$ and the ordinates x = 2 and x = 4 If the ordinate at x = a divides the area into two equal parts find 'a'

Ans: Note 2<a<4 a = $2\sqrt{2}$

3) Find the area included between the parabolas $y^2=4ax$ and $x^2=4ay$ Ans: $16\frac{a^3}{3}$ sq.units.

4) Find the area of the segment cut off from the parabola $y^2=2x$ by the line y=4x-1

Ans: $\frac{9}{32}$ sq.units

5) Show that the area enclosed by the circle $x^2+y^2 = 64a^2$ and the parabola $y^2= 12ax$ is $a^2\left(\frac{16}{\sqrt{3}} + \frac{64\Pi}{\sqrt{3}}\right)$

6) Sketch the region bounded by the curves $y = \sqrt{5 - x^2}$ and $y = |x - 1|$ and find its area.

Ans: $\frac{5}{2}\left[\text{Sin}^{-1}\frac{2}{\sqrt{5}} + \text{Sin}^{-1}\frac{1}{\sqrt{5}}\right] - \frac{1}{2}$

7) Find the area of the region bounded by the curve C. y = tanx the tangent drawn to C at $x = \Pi/4$ and the x-axis.

Ans: $\frac{1}{2}\left(\log 2 - \frac{1}{2}\right)$

8) Find the area of the region lying above x-axis and included between the curves $x^2+y^2 = 2ax$ and $y^2 = ax$

Ans: $a^2\left(\frac{\Pi}{4} - \frac{2}{3}\right)$

9) Sketch the region bounded by the curves $y = x^2$ and $y = \frac{2}{1+x^2}$ and find its area.

Ans: $\Pi - \frac{2}{3}$

10) Find the area of the smaller region bounded by the curve $\frac{x^2}{16} + \frac{y^2}{9} = 1$ and the straight line $\frac{x}{4} + \frac{y}{3} = 1$

Ans: $\frac{\Pi}{3}$ sq.units.

11) Using integration find the area of the triangle ABC where A is (2,3) B(4,7) and C(6,2)

Ans: 4 sq.units.

12) Using integration find the area of the triangle ABC whose vertices are A(3,0), B(4,6) and C(6,2)

Ans: 8 sq.units.

13) Find the area included between the curves $(x-1)^2+y^2 = 1$ and $x^2+y^2 = 1$

Ans: $\left(\frac{2\Pi}{3} - \frac{\sqrt{3}}{2}\right)$ sq.units.

14) Sketch the region common to the circle $x^2+y^2 = 25$ and the parabola $y^2 = 8x$ also find the area of the region using integration.

Ans : $\left\{\frac{\sqrt{2}}{3}\left(\sqrt{41}-4\right)^{3/2} + \frac{25}{4}\Pi - \frac{25}{2}\sin^{-1}\left(\frac{\sqrt{41}-4}{5}\right)\right\}$

15) Find the area of the circle $x^2 + y^2 = a^2$ Ans: Πa^2 sq.units.

16) Sketch the region of the ellipse and find its area using integration. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \quad a>b$

Ans: Πab sq.units.

17) Find the area of the region given by : $\{(x,y): x^2 \le y \le |x|\}$ Ans: $\frac{1}{3}$ sq.units

18) Find the area of the region

$\{(x,y): y^2 \le 4x, 4x^2+4y^2 = 9\}$ Ans: $\left\{\frac{\sqrt{2}}{6}+\frac{9\Pi}{8}-\frac{9}{4}\sin^{-1}\frac{1}{3}\right\}$ sq.units.

19) Find the area of the region bounded by the circle $x^2+y^2 = 16$ and the line y = x in the first quadrant. Ans: 2Π sq.units.

20) Find the area of the smaller region bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the straight line $\frac{x}{a}+\frac{y}{b}=1$ Ans: $\frac{ab}{4}(\Pi-2)$ sq.units.

21) Find the area bounded by the curve y = sinx, x=axis and between x = 0 , x = Π

Ans: 2 sq.units.

22) Sketch the graph of y = |x-1| and evaluate $\int_{-2}^{4}|x-1|dx$ Ans: 9 sq.units.

23) Find the area of the region enclosed between the circles $x^2+y^2 = 1$ and $\left(x-\frac{1}{2}\right)^2+y^2=1$

Ans: $\left(\frac{-2\sqrt{3}+\sqrt{15}}{16}-2\sin^{-1}\frac{1}{4}+\Pi\right)$ sq.units.

24) Draw the rough sketch of y = sin2x and determine the area enclosed by the lines $x = \Pi/4$ and $x = \frac{3\Pi}{4}$

Ans: 1 sq.units.

25) Compute the area bounded by the lines x+2y = 2, y-x = 1 and 2x+y = 7.

Ans: 6 sq.units.

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HOTS : DIFFERENTIAL EQUATIONS

1) Write the order and degree of the differential equation. $\left[1+\left(y^{I}\right)^{2}\right]^{3/2} = y^{II}$

(Ans: 2,2)

2) Form the differential equation of the family of curves represented by the equation

$(x+a)^2 - 2y^2 = a^2$ (Ans: $x^2 + 2y^2 - 4xyy^{I} = 0$)

3) Write the order and degree of the diff. equation $\left(\frac{d^2y}{dx^2}\right)^{2/3} = \left(y + \frac{dy}{dx}\right)^{1/2}$

(Ans: 2,4)

4) Form the differential equation of circles represented by

$(x-\alpha)^2 + (y-\beta)^2 = r^2$ by eliminating α and β Ans: $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = r^2\left(\frac{d^2y}{dx^2}\right)^2$

5) Show that $Ax^2+By^2 = 1$ is a solution of the diff. equation. $x\left[y\left(\frac{d^2y}{dx^2}\right)+\left(\frac{dy}{dx}\right)^2\right] = y\frac{dy}{dx}$

6) Solve $(x+y+1)\frac{dy}{dx} = 1$ $[\text{Ans}: x = ce^y - y - 2]$

7) Solve : $(x+y)^2\frac{dy}{dx} = a^2$ $\left[\text{Ans}: (x+y) - a\tan^{-1}\left(\frac{x+y}{a}\right) = x + c\right]$

8) Solve : $x\frac{dy}{dx} = y(\log y - \log x + 1)$ $[\text{Ans}: y = xe^{cx}]$

9) Solve : $ydx - (x+2y^2)dy = 0$ $\left[\text{Ans}: \frac{x}{y} = 2y + c\right]$

10) Solve : $\left[x\sqrt{x^2+y^2} - y^2\right]dx + xydy = 0$ $\left[\text{Ans}: \sqrt{x^2+y^2} + \log\frac{y}{x} = Cx\right]$

11) Solve : $\frac{dy}{dx} = \cos^3 x \sin^4 x + x\sqrt{2x+1}$ $\left[\text{Ans}: y = \frac{1}{5}\sin^5 x - \frac{1}{7}\sin^7 x + \frac{1}{10}(2x+1)^{5/2} - \frac{1}{6}(2x+1)^{3/2} + c\right]$

12) Solve : $(x^3y^3 + xy)\frac{dy}{dx} = 1$ $\left[\text{Ans}: e^{y^{3/2}}\left(-\frac{1}{x}\right) = y^2e^{y^{5/2}} - 2e^{y^{3/2}} + c\right]$

13) Solve : $(x^3 - 3xy^2)dx = (y^3 - 3x^2y)dx$ $[\text{Ans}: x^2 - y^2 = C(x^2+y^2)^2]$

14) Solve : $\sqrt{1-y^2}\,dx = [\sin^{-1} y - x]dy$ $\left[\text{Ans}: x = (\sin^{-1}y - 1) + Ce^{\sin^{-1}y}\right]$

15) Solve the differential equation :

$(x^2y + y^2x)dy = (x^3 + y^3)dx$ $\left[\text{Ans}: -\frac{y}{x} = \log(x-y) + c\right]$

16) Solve : $\frac{dy}{dx} = \frac{y^2+y+1}{x^2+x+1} = 0$ $[Ans : (x+y+1) = C(1-x-y-2xy)]$

17) Solve : $(x^3 - 3xy^2)dx = (y^3 - 3x^2y)dy$ $[Ans : x^2 - y^2 = c(x^2 + y^2)^2]$

18) Solve : $\frac{dy}{dx} = \frac{x^2+y^2+1}{2xy}$, where $y(1) = 1$ $[Ans : y^2 = x^2 + x - 1]$

19) Solve : $\frac{xdx + ydy}{xdy - ydx} = \sqrt{\frac{a^2 - x^2 - y^2}{x^2 + y^2}}$ $\left[Ans : \sqrt{x^2+y^2} = a\sin\left(c + \tan^{-1}\frac{y}{x}\right)\right]$

(Hint : take $x = r\cos\theta$, $y = r\sin\theta$, so that $x^2 + y^2 = r^2$]

20) When the interest is compounded continuously, the amount of money invested increases at a rate proportional to its size. If Rs.1000 is invested at 10% compounded continuously, in how many years will the original investment double itself?

$[Ans : 10\log_e^2 \text{ years}]$

21) A population grows at the rate of 8% per year. How long does it take for the population to double?

$\left[Ans : \frac{25}{2}\log 2 \text{ years}\right]$

22) A wet porous substance in the open air losses its moisture at a rate proportional to the moisture content. If a sheet hung in the wind loses half of its moisture during the first hour, when will it have lost 95% moisture, weather conditions remaining the same.

$\left[Ans : \frac{\log 20}{\log 2}\right]$

23) The surface area of a balloon being inflated, changes at a rate proportional to time t. If initially its radius is 1 unit and after 3 seconds it is 2 units, find the radius after time 't'.

$\left[Ans : r = \sqrt{1 + \frac{t^2}{3}}\right]$

24) A curve passing through the point (1,1) has the property that the perpendicular distance of the origin from the normal at any point P of the curve is equal to the distance of p from the x-axis. Determine the equation of the curve.

$[Ans : x^2 + y^2 = 2x]$

25) Newton's law of cooling states that the rate of change of the temperature T of an object is proportional to the difference between T and the (constant) temperature t of the surrounding medium. We can write it as

$$\frac{dT}{dt} = -k(T - t),\ k > 0 \text{ constant.}$$

A cup of coffee is served at 185°F in a room where the temperature is 65°F. Two minutes later the temperature of the coffee has dropped to 155°F. (log 3/4=0.144, log3 = 1.09872). Find the time required for coffee to have 105°F temperature.

(Ans: 7.63 min.]

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HOTS : VECTORS

1) Find the unit vector perpendicular to both the vectors

$\bar{a} = 4\bar{i} - \bar{j} - 3\bar{k}$ and $\bar{b} = 2\bar{i} + 2\bar{j} - \bar{k}$ Ans : $\frac{1}{3}\left(-\bar{i} + 2\bar{j} + 2\bar{k}\right)$

2) If $\bar{\alpha} = 3\bar{i} - \bar{j}$ and $\bar{\beta} = 2\bar{i} + \bar{j} - 3\bar{k}$. Express $\bar{\beta}$ as a sum of two vectors $\bar{\beta}_1$ & $\bar{\beta}_2$, where $\bar{\beta}_1$ is parallel to $\bar{\alpha}$ and $\bar{\beta}_2$ is perpendicular to $\bar{\alpha}$.

3) If $\bar{a} + \bar{b} + \bar{c} = 0$, show that $\bar{a} x \bar{b} = \bar{b} x \bar{c} = \bar{c} x \bar{a}$

4) Prove the triangle inequality $\left|\bar{a} x + \bar{b} \le |\bar{a}|\right| + |\bar{b}|$

5) Prove Cauchy - Schawarz inequality : $\left(\bar{a}.\bar{b}\right)^2 \le |\bar{a}|^2.|\bar{b}|^2$

6) If $\bar{a}$ and $\bar{b}$ are vectors, prove that $|\bar{a} x \bar{b}|^2 + \left(\bar{a}.\bar{b}\right)^2 = |\bar{a}|^2.|\bar{b}|^2$

7) Prove that angle in a semi-circle is a right angle.

8) If $\hat{a}$ and $\hat{b}$ are unit vectors inclined at an angle θ, then prove that

a) $\cos\frac{\theta}{2} = \frac{1}{2}|\hat{a} + \hat{b}|$ b) $\tan\frac{\theta}{2} = \frac{|\hat{a} - \hat{b}|}{|\hat{a} + \hat{b}|}$

9) Show that the perpendicular of the point $\bar{c}$ from the line joining $\bar{a}$ & $\bar{b}$ is $\frac{|\bar{a} x \bar{b} + \bar{b} x \bar{c} + \bar{c} x \bar{a}|}{|\bar{b} - \bar{a}|}$

(Hint : use area of triangle = $\frac{1}{2}bh$)

10) Show that the area of the parallelogram having diagonals $3\bar{i} + \bar{j} - 2\bar{k}$ and $\bar{i} - 3\bar{j} + 4\bar{k}$ is $5\sqrt{3}$

11) Vectors $2\bar{i} - \bar{j} + 2\bar{k}$ and $\bar{i} + \bar{j} - 3\bar{k}$ act along two adjacent sides of a parallelogram. Find the angle between the diagonals of the parallelogram.

12) L and M are the mid-points of sides BC & DC of a paralellogram ABCD. Prove that

$\overline{AL} + \overline{AM} = \frac{3}{2}\overline{AC}$

13) Let $\bar{a}, \bar{b}$ & $\bar{c}$ be three vectors such that $|\bar{a}| = 3, |\bar{b}| = 4, |\bar{c}| = 5$ and each one of them being perpendicular to sum of the other two, find $|\bar{a} + \bar{b} + \bar{c}|$ [Ans : $5\sqrt{2}$]

14) Prove that the area of a paralellogram with diagonals $\bar{a}$ and $\bar{b}$ is $\frac{1}{2}|\bar{a} x \bar{b}|$

15) If $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are the p^{th}, q^{th} and r^{th} terms of an AP and

$\bar{u} = (q - r)\bar{i} + (r - p)\bar{j} + (p - q)\bar{k}$ & $\bar{v} = \frac{1}{a}\bar{i} + \frac{1}{b}\bar{j} + \frac{1}{c}\bar{k}$ then prove that $\bar{u}$ & $\bar{v}$ are orthogonal vectors.

16) In a triangle ABC, prove that $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$

17) Using vector method prove that :

a) sin(A+B) = sinAcosB + cosAsinB

b) sin(A-B) = sinAcosB — cosAsinB

c) cos(A+B) = cosAcosB — sinAsinB

d) cos(A-B) = cosAcosB + sinAsinB

18) Using vector method, show that the angle between two diagonals of a cube is $\cos^{-1}\left(\frac{1}{3}\right)$

19) Prove that the altitudes of a triangle are concurrent.

20) Prove that the perpendicular bisectors of a triangle are concurrent.

21) Using vector method, prove that if the diagonals of a parallelogram are equal in length, then it is a rectangle.

22) Using vector method, prove that if two medians of a triangle are equal, then it is an isosceles.

23) Using vector method, show that the diagonals of a Rhombus bisect each other at right angles.

24) Prove by vector method, that the parallelogram on the same base and between the same parallels are equal in area.

25) If a, b & c are the lengths of the sides opposite respectively to the angles A, B & C of a ΔABC, using vector method show that

a) $\cos A=\frac{b^2+c^2-a^2}{2bc}$ b) $a=b\cos C+C\cos B$

26) If D, E & F are the mid-points of the sides of a triangle ABC, prove by vector method that area of

$\Delta DEF=\frac{1}{4}(\text{area of } \Delta ABC)$

27) If a, b & C are the lengths of the sides of a triangle, using vector method, show that its area is $\sqrt{s(s-a)(s-b)(s-c)}$

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HOTS : 3D GEOMETRY

1) Show that the line $\bar{r} = (2\bar{i} - 2\bar{j} + 3\bar{k}) + \lambda(\bar{i} - \bar{j} + 4\bar{k})$ is parallel to the plane $\bar{r}.(\bar{i} + 5\bar{j} + \bar{k}) = 5$

2) Find the equation of the plane passing through the point (1,4,-2) and parallel to the plane —2x+y—3z=7 [Ans:2x—y+3z+8=0]

3) What are the conditions that the planes $a_1x+b_1y+c_1z = d_1$ & $a_2x+b_2y+c_2z = d_2$ are (i) parallel (ii) perpendicular to each other?

Ans: (i) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ (ii) $a_1a_2 + b_1b_2 + c_1c_2 = 0$

4) Find the value of k for which the two lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = z$ intersect at a point? $\left[\text{Ans}: k = \frac{9}{2}\right]$

5) Find the equation of the plane passing through the point (1,6,3) and perpendicular to the plane 2x+3y—z = 7 (Ans: 3x+y+9z = 36)

6) Find the value of k for which the lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$ are co-planar. [Ans: k = 0, —3}

7) Find the equation of the plane passing through the point (1,1,1) and perpendicular to each of the plane x+2y+3z = 7 and 2x—3y+4z = 0 [Ans: 17x+2y—7z—12 = 0]

8) Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = 2$ intersect. Also find the point of intersection. [Ans: (—1,—1,—1]

9) Find the image of the point (1,2,3) in the plane x+2y+4z = 38. [Ans: (3,6,11)]

10) Find the equation of the plane passing through the points (1,—1,2) and (2,—2,2) and perpendicular to the plane 6x—2y+2z = 9. [Ans: x+y—2z+4 = 0]

11) Find the foot of the perpendicular drawn from the point A(1,0,3) to the join of the points B(4,7,1) and C(3,5,3) $\left[\text{Ans}: \frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right]$

12) Find the length and co-ordinates of the foot of perpendicular from point (1,1,2) to the plane 2x—2y+4z+5 = 0

$$\left[\text{Ans}: \frac{13\sqrt{6}}{12}, \left(-\frac{1}{12}, \frac{25}{12}, -\frac{1}{6}\right)\right]$$

13) Find the equation of the plane through the points (—1,1,1) and (1,—1,1) perpendicular to the plane x+2y+2z = 5 (Ans: 2x+2y—3z+3 = 0)

14) Find the perpendicular distance of point (2,3,4) from the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$

$$\left[\text{Ans}: \frac{1}{49}\sqrt{44541}\right]$$

15) The foot of the perpendicular drawn from the origin to the plane is (2,5,7). FInd the equation of the plane. [Ans: 2x+5y+7z = 78]

16) Find the values of P so that the lines $\frac{1-x}{3}=\frac{7y-14}{2p}=\frac{z-3}{2}$ and $\frac{7-7x}{3p}=\frac{y-5}{1}=\frac{6-z}{5}$ are at right angles. $\left[\text{Ans}: \frac{70}{11}\right]$

17) Find the shortest distance between two lines whose vector equations are

$\bar{r}=(1-t)\bar{i}+(t-2)\bar{j}+(3-2t)\bar{k}$ and $\bar{r}=(s+1)\bar{i}+(2s-1)\bar{j}-(2s+1)\bar{k}$ $\left(\text{Ans}: \frac{8}{\sqrt{29}}\right)$

18) Find the vector equation of the plane through the intersection of the planes

$\bar{r}.(2\bar{j}+6\bar{j})+12=0$ & $\bar{r}.(3\bar{i}-\bar{j}+4\bar{k})=0$ which are at a unit distance from the origin.

$\left[\text{Ans}: \bar{r}.\left(-\bar{i}+2\bar{j}-2\bar{k}\right)+3=0\right]$

19) Find the equation of the line passing through the point (3,0,1) and parallel to the planes x+2y = 0 and 3y –z = 0 $\left[\text{Ans}: \bar{r}=(3\bar{i}+\bar{k})+\lambda(-2\bar{i}+\bar{j}+3\bar{k})\right]$

20) Find the reflection of the point (1,2,–1) in the plane 3x–5y+4z = 5 $\left[\text{Ans}: \left(\frac{73}{25},\frac{-6}{5},\frac{39}{25}\right)\right]$

21) Find the distance of the point (1,–2,3) from the plane x–y+z = 5 measured parallel to the line $\frac{x+1}{2}=\frac{y+3}{2}=\frac{z+1}{-6}$ [Ans: 1]

22) Find the distance of the point (2,3,4) from the line $\frac{x+3}{3}=\frac{y-2}{6}=\frac{z}{2}$ measured parallel to the plane 3x+2y+2z+5 = 0

23) A line makes angles α,β,γ and δ with the four diagonals of a cube. Prove that

$\cos^2\alpha+\cos^2\beta+\cos^2\gamma+\cos^2\delta=\frac{4}{3}$

24) A variable plane which remains at a constant distance 3p from the origin cuts the co-ordinate axes at A, B & C. Show that the locus of the centroid of the ΔABC is $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{1}{p^2}$

25) A variable plane is at constant distance p from the origin and meet the axes in A, B & C. Show that the locus of the centroid of the tetrahedron ΔABC is $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{16}{p^2}$

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LINEAR PROGRAMMING

4 Marks/6 marks

1) A toy company manufactures two types of doll; a basic version-doll A and a deluxe version doll B. Each doll of type B takes twice as long as to produce as one of type A, and the company would have time to make a maximum of 2000 per day if it produces only the basic version. The supply of plastic is sufficient to produce 1500 dolls per day (both A and B combined). The deluxe version requires a fancy dress of which there are only 600 per day available. If company makes profit of Rs.3 and Rs.5 per doll, respectively, on doll A and B; how many each should be produced per day in order to maximize profit.

Ans: Type A = 1000, Type B = 500, Max. profit = Rs.5500

2) A dietician has to develop a special diet using two foods P and Q. Each packet (containing 30 g) of food P contains 12 units of calcium, 4 units of iron, 6 units of cholesterol and 6 units of vitamin A, while each packet of the same quality of food Q contains 3 units of calcium, 20 units of vitamin A. The diet requires atleast 240 units of calcium, atleast 460 units of iron and almost 300 units of cholesterol. How many packets of each food should be used to maximize the amount of vitamin A in the diet? What is the maximum amount of vitamin A?

Ans: 40 packets of food P and 15 packets of food Q Maximum at (40,15) = 285

3) An oil company has tow depots A and B with capacities of 7000L and 4000L respectively. The company is to supply oil to three petrol pumps D, E and F whose requirements are 4500L, 3000L and 3500L respectively. The distances (in km) between the depots and the petrol pumps is given in the following table:

	Distance in km	
From/To	A	B
D	7	3
E	6	4
F	3	2

Assuming that the transportation cost of 10 litres of oil is Rs.1 per km, how should the delivery be scheduled in order that the transportation cost is minimum.

Ans: From A : 500, 3000 and 3500 litres, From B : 4000, 0, 0 litres to D, E and F respectively. Minimum cost = Rs.4400

4) A firm makes two types of furniture : chairs and tables. The contribution to profit for each product as calculated by the accounting department is Rs.20 per chair and Rs.30 per table. Both products are to be processed on three machines M_1, M_2 and M_3. The time required in hours by each product and total time available in hours per week on each machine are as follows:

Machine	Chair	Table	Available Time
M_1	3	3	36
M_2	5	2	50
M_3	2	6	60

How should the manufacturer schedule his production in October to maximize profit.

Ans: 3 chairs and 9 tables.

5) A farmer has a supply of chemical fertilizer of type I which contains 10% nitrogen and 6% phosphoric acid and type II fertilizer which contains 5% nitrogen and 10% phosphoric acid. After testing the soil conditions of a field, it is found that atleast 14 kg of nitrogen and 14 kg of phosphoric acid is required for a good crop. The fertilizer type I costs Rs.2.00 per kg and type II costs Rs.3.00 per kg. How many kilograms of each fertilizer should be used to meet the requirement and the cost be minimum.

Ans: Minimum at (100,80) and is equal to Rs.440.

6) If a young man rides his motorcycle at 25 km/hr, he had to spend Rs.2 per km on petrol. If he rides at a faster speed of 40 km/hr, the petrol cost increases at Rs.5 per km. He has Rs.100 to spend on petrol and wishes to find what is the maximum distance he can travel within one hour. Express this as LPP and solve it graphically.

Ans: Maximum at $\left(\frac{50}{3}, \frac{40}{3}\right)$ and is equal to 30 km.

7) Solve the following LPP graphically. Maximize or minimize Z = 3x+5y subject to

$$3x - 4y \geq -12$$
$$2x - y + 2 \geq 0$$
$$2x + 3y - 12 \geq 0$$
$$0 \leq x \leq 4$$
$$y \geq 2$$

Ans: Min. value 19 at (3,2) and Max. value 42 at (4,6)

8) Solve the following LPP graphically. Minimize Z = 3x+5y subject to

$$-2x + y \leq 4$$
$$x + y \geq 3$$
$$x - 2y \leq 2$$
$$x, y \geq 0$$

Ans : Minimum value is $\frac{29}{3}$ at $\left(\frac{8}{3}, \frac{1}{3}\right)$

9) Determine graphically the minimum value of the objective function.

$$Z = -50x + 20y$$

Subject to constraints

$$2x - y \geq -5$$
$$3x + y \geq 3$$
$$2x - 3y \leq 12$$
$$x \geq 0,\ y \geq 0$$

10) Find the maximum and minimum values of 5x+2y subject to constraints

$$-2x - 3y \leq -6$$
$$x - 2y \leq 2$$
$$6x + 4y \leq 24$$
$$-3x + 2y \leq 3$$
$$x \geq 0 \text{ and } y \geq 0$$

Ans : Max. value is 19 at $\left(\frac{7}{2}, \frac{3}{4}\right)$ and

Min. value is 4.85 at $\left(\frac{3}{13}, \frac{24}{13}\right)$

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HOTS : HIGH ORDER THINKING QUESTIONS - MATHEMATICS
PROBABILITY

1) Urn A contain 1 white, 2 black, 3 red balls. Urn B contain 2 white, 1 black, 1 red balls. Urn C contains 4 white, 5 black, 3 red balls. Two balls are drawn from one of the Urn and found to be one white and one red. Find the probabilities that they come from Urns A, B or C.

16/39

2) A die is thrown 120 times and getting 1 or 5 is considered success. Find the mean, variance of number of successes. $\mu = 40, \sigma^2 = 26.7$

3) Given P(A) = 0.3, P(B) = 0.2, Find P(B/A) if A and B are mutually exclusive events. (0)

4) The parameters n and p are 12 and $\frac{1}{3}$ for a binomial distribution. Find standard deviation.

(1.63)

5) A man fires 4 bullets on a thief. The probability that the thief will be killed by one bullet is 0.6. Find the probability that the thief is still alive. $(0.4)^4$

6) In a hurdle race a player has to cross 10 hurdles. The probability that will clear each hurdle is $\frac{5}{6}$, what is the probability that he will knock down fewer than 2 hurdles. (0.4845

7) If on an average 1 ship in every 10 sinks, find the chance that out of 5 ships atleast 4 will arrive safely. (0.9185)

8) 4 persons are chosen at random from a group of 3 men, 2 women, 3 children. Find the probability that out of 4 choice, exactly 2 are children. $\frac{3}{7}$

9) Suppose X has a binomial distribution B (6, $\frac{1}{2}$) show that X = 3 is the most likely outcome.

10) In a binomial distribution, the sum of mean and variance is 42. Product is 360. FInd the distribution.

$$\left(\frac{2}{5} + \frac{3}{5}\right)^{50}$$

11) Given that the two numbers appearing on two dice are different. Find the probability of the event 'the sum of numbers on the dice is 4'. $\frac{1}{15}$

12) $P(A) = \frac{1}{2}, P(B) = \frac{7}{12}$ $P(\text{not A or not B}) = \frac{1}{4}$ state whether A and B are independent. (No)

13) Three cards are drawn with replacement from a well shuffled pack of cards. Find the probability that cards are a king, queen and a jack. $\frac{6}{2197}$

14) Find the probability of throwing almost 2 sixes in 6 throws of a single dice. $\left(\frac{35}{18}\right)\left(\frac{5}{6}\right)^4$

15) Find the probability that sum of the numbers showing on the two dice is 8, given that atleast one dice doesn't show five. $\left(\frac{3}{25}\right)$

16) The mean and variance of a binomial distribution are 4 and $\frac{4}{3}$. Find $P(X \geq 1)$ $\left(\frac{728}{729}\right)$

17) 6 boys and 6 girls sit in a row at random. Find the probability that 1) The six girls sit together 2) The boys and girls sit alternatively. $\left(\frac{1}{132}\right)\left(\frac{1}{462}\right)$

18) If A, B, C are events associated with random expt. Prove that

$$PtP(A \cup B \cup C) = P(A) + P(B) + P(c) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

19) Shreya visits the cities A, B, C and D at random. What is the probability that he visits

1) A before B 2) A just before B. $\left(\frac{1}{2}, \frac{5}{24}\right)$

20) What are the odds in favour of getting a '3' in a throw of die? What are the odds against getting 3? $\left(\frac{1}{5}, \frac{5}{1}\right)$

21) A pack of 52 cards were distributed equally among 4 players. Find the chance that 4 kings are held by particular player. $\left(\frac{11}{4165}\right)$

22) A fair die is rolled. The probability that the first 1 occurs at the even number of trails is $\left(\frac{5}{11}\right)$

23) If a 4 digit number > 5000 are randomly formed from digits 0,1,3,5,7. Find probability of forming a number divisible by 5 when (1) digits are repeated 2) digits are not repeated.

$\left(\frac{2}{5}\right)\left(\frac{3}{8}\right)$

24) A letter is chosen at random from the word "ASSASSINATION". Find the probability that the letter is a vowel. $\left(\frac{6}{13}\right)$

25) A fair die is rolled. The probability that the first 1 occurs at even number of trails is $\left(\frac{5}{11}\right)$

26) If three distinct numbers are chosen at randomly from first 100 natural nos. then the probability that all of them are divisible by 2 or 3 is $\left(\frac{4}{1155}\right)$

27) A coin is tossed 7 times. FInd the probability distribution of getting 'r' heads.

$\left(7c_r\left(\frac{1}{2}\right)^7, r = 0,1,2......7\right)$

28) A company produces 10% defective items. Find the probability of getting 2 defective items in a sample of 8 items is $\frac{28x9^6}{10^8}$

29) Obtain the probability distribution of number of sixes in two tosses of a dice. Also find mean/ variance. $\left(25/36, 10/36, 1/36, 1/3, 5/18\right)$

30) A,B,C tosses a coin in turns. The first one to throw a 'head' wins game. What are their respective chances of winning. $\left(4/7, 2/7, 1/7\right)$

31) A man is known to speak truth 3 times out of 4. He throws a dice and reports that it is a six. Find the probability that it is actually a six. $\left(3/4\right)$

32) Suppose that a fair dice are tossed and let X represents "The sum of points". FInd the mean/ variance of X. $\left(7, \sqrt{2.41}\right)$

33) Find the probability that sum of nos. appearing and showing on two dice is 8, given that atleast one of the dice doesn't show 5. $\left(1/9\right)$

34) A tells lie is 30% cases, and B in 35% cases find the probability that both state same fact.

35) Two cards are drawn without replacement from a pack. Find the probability distribution of number of face cards. $\left[\frac{105}{221}, \frac{96}{221}, \frac{20}{221}\right]$

P(X) = 0 1 2

36) A man takes a step forward with probability 0.4 and backwards with a probability 0.6. Find the probability that after 11 steps he is just one step away from the starting point.

$(462) \times (0.24)^5$

37) Find the probability distribution of the sum of the numbers obtained when two dice are thrown once. (All 11 prob. distributions to be shown)

38) Two cards are drawn from a pack. Find the probability that number of aces are drawn. (Write probability distribution table)

39) Find the mean and variance of number of sines in two tosses of a die. $\left(\frac{1}{3}\right), \left(\frac{5}{18}\right)$

40) 6 coins are tossed simultaneously. Find the probability of getting 1) no heads 2) 3 heads

$\left(\frac{1}{64}\right), \left(\frac{5}{16}\right)$

41) If $2p(A) = P(B) = \frac{5}{13}$ and $P\left(\frac{A}{B}\right) = \frac{2}{5}$ Find $P(A \cap B)$

42) If E and F are events such that $P(F) = \frac{1}{4}$ $P(F) = \frac{1}{2}$ and $P(E \cap P) = \frac{1}{8}$ find $P(\bar{E} \cap \bar{F})$

43) P(A speaks truth is) = $\frac{4}{5}$. A coin is tossed. 'A' reports that a head appears. The probability that actually there was a head is.

44) Two cards are drawn from a pack and kept out. Then one card is drawn from remaining 50 cards. Find the prob. that it is an ace. $\left(\frac{1}{13}\right)$

45) Two dice are thrown. Find the probability that the number appeared have a sum 8 if it is known that second dice always exhibits 4. $\left(\frac{1}{6}\right)$

46) If the second die always shows an odd no. find the conditional probability of getting a sum as 7, if a pair of dice is to be known. $\left(\frac{1}{6}\right)$

47) $P(A) = \frac{1}{2}, P(B) = \frac{7}{12}$ and P(not A or not B) = $\frac{1}{4}$. State whether A and B independent or not. (NO)

48) A die is thrown again and again untill three sixes are obtained. Find the probability of obtaining third six in sixth throw of dice. $\left(\frac{625}{23328}\right)$

49) Six dice are thrown 729 times. How many times do you expect atleast three dice to show 5 or 6.

(233)

50) A random variable X has probability distribution P(X) of the following form where K is some number.

$$P(X) = \begin{cases} k & \text{if} \quad x = 0 \\ 2k & \text{if} \quad x = 1 \\ 3k & \text{if} \quad x = 2 \\ 0 & \quad \text{otherwise} \end{cases}$$

$\left(\frac{1}{6}\right)\left(\frac{1}{2}\right)$

Find (1) K (2) P (X > 2)

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