Points to Remember:

1. **Cuboid**
   - (i) Volume = \(lbh\)
   - (ii) Curved surface area = \(2h (l + b)\)
   - (iii) Total surface area = \(2 (lb + bh + lh)\)
   - (iv) Diagonal = \(\sqrt{l^2 + b^2 + h^2}\)

2. **Cube**
   - (i) Volume = \(a^3\)
   - (ii) Curved surface area = \(4a^2\)
   - (iii) Total surface area = \(6a^2\)
   - (iv) Diagonal = \(\sqrt{3}a\)

3. **Cylinder**
   - (i) Volume = \(\pi r^2 h\)
   - (ii) Curved surface area = \(2\pi rh\)
   - (iii) Total surface area = \(2\pi r (r + h)\)

4. **Hollow Cylinder**
   - (i) Volume = \(\pi h (R^2 - r^2)\)
   - (ii) Curved surface area = \(2 \pi h (R + r)\)
   - (iii) Total surface area = \(2\pi h (R + r) + 2\pi (R^2 - r^2)\)
     \[= 2\pi (R + r) (h + R - r)\]
5. **Cone**

   (i) Volume \( V = \frac{1}{3} \pi r^2 h \)

   (ii) slant height, \( l = \sqrt{h^2 + r^2} \)

   (iii) curved surface area = \( \pi rl \)

   (iv) Total surface area = \( \pi r(l + r) \)

6. **Sphere**

   (i) Volume \( V = \frac{4}{3} \pi r^3 \)

   (ii) Total surface area = \( 4 \pi r^2 \)

7. **Spherical Shell**

   (i) Volume \( V = \frac{4}{3} \pi (R^3 - r^3) \)

   (ii) Surface area (outer) = \( 4 \pi R^2 \)

8. **Hemi-sphere**

   (i) Volume \( V = \frac{2}{3} \pi r^3 \)

   (ii) Curved surface area = \( 2 \pi r^2 \)

   (iii) Total surface area = \( 3 \pi r^2 \)

9. **Frustum of a cone**:

   (i) Volume \( V = \frac{1}{3} \pi h (r^2 + R^2 + rR) \)

   (ii) Slant height \( l = \sqrt{h^2 + (R - r)^2} \)

   (iii) Curved surface area = \( \pi l (r + R) \)

   (iv) Total surface area = \( \pi d (r + R) + \pi (r^2 + R^2) \)
ILLUSTRATIVE EXAMPLES

Example 1. A toy is in form of a cone mounted on a hemisphere of radius 3.5 cm. The total height of the toy is 15.5 cm. Find its total surface area.

Solution. Here, \( AO = AC - OC = 15.5 \text{ cm} - 3.5 \text{ cm} = 12 \text{ cm} \)

Now, in \( \triangle AOB \), \( AB^2 = AO^2 + OB^2 \)

\[ = (12)^2 + (3.5)^2 \]
\[ = 144 + 12.25 \]
\[ = 156.25 \]

\( \Rightarrow AB = \sqrt{156.25} = 12.5 \text{ cm} \)

Total curved surface area = curved surface area of the cone + curved surface area of the hemisphere

\[ = \pi r l + 2\pi r^2 = \pi (l + 2r) \]

\[ = \frac{22}{7} \times \frac{7}{2} \times [12.5 + 2(3.5)] \text{ cm}^2 \]

\[ = 214.5 \text{ cm}^2 \text{ Ans.} \]

Example 2. A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter \( l \) of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.

Solution. Here, \( r = \frac{l}{2} \), edge of a cube = \( l \)

Total surface area of the cube after hemispherical depression

\[ = \text{TSA of a cube} - \text{base area of a hemisphere} + \text{CSA of a hemisphere} \]

\[ = 6l^2 - \pi r^2 + 2\pi r^2 = 6l^2 + \pi r^2 \]

\[ = 6l^2 + \pi \left( \frac{l}{2} \right)^2 = 6l^2 + \frac{l^2}{4} \pi \]

\[ = \frac{l^2}{4}(\pi + 24) \text{ sq. units Ans.} \]

Example 3. From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the surface area of the remaining solid to the nearest \( \text{cm}^2 \).

Solution. For a conical part, we have \( r = \frac{14}{20} \text{ cm} = 0.7 \text{ cm}, h = 2.4 \text{ cm} \)

\( \Rightarrow \) slant height of a cone \( (l) = \sqrt{h^2 + r^2} = \sqrt{(2.4)^2 + (0.7)^2} \text{ cm} \)

\[ = \sqrt{5.76 + 0.49} \text{ cm} = \sqrt{6.25} \text{ cm} = 2.5 \text{ cm} \]

Total surface area of the solid left after conical cavity

\[ = \text{TSA of a cylinder} - \text{area of the base of a cone} + \text{CSA of the cone} \]

\[ = (2\pi rh + 2\pi r^2) - \pi r^2 + \pi rl \]

\[ = 2\pi rh + \pi r^2 + \pi rl \]
\[ = \pi r(2h + r + l) \]
\[ = \frac{22}{7} \times \frac{7}{10} \times [2(2.4) + 0.7 + 2.5] \text{ cm}^2 \]
\[ = 2.2 \times 8 \text{ cm}^2 = 17.6 \text{ cm}^2 \approx 18 \text{ cm}^2 \text{ (approx) Ans.} \]

**Example 4.** A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in the given figure. If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm, find the total surface area of the article.

**Solution.** For a given solid, we have

\[ r = \frac{7}{2} \text{ cm, } h = 10 \text{ cm} \]

Total surface area of the solid by scooping out a hemisphere from each end of a cylinder

\[ = \text{CSA of the cylinder} + 2 \times (\text{CSA of the hemisphere}) \]
\[ = 2\pi rh + 2(2\pi r^2) \]
\[ = 2\pi(r + 2r) \]
\[ = 2\times \frac{22}{7} \times \frac{7}{2} \left[ 10 + 2 \times \left( \frac{7}{2} \right) \right] \text{ cm}^2 \]
\[ = 22 \times (10 + 7) \text{ cm}^2 = 22 \times 17 \text{ cm}^2 = 374 \text{ cm}^2 \text{ Ans.} \]

**Example 5.** A tent is of the shape of a right circular cylinder up to a height of 3 metres and then becomes a right circular cone with a maximum height of 13.5 metres above the ground. Calculate the cost of painting the inner side of the tent at the rate of Rs. 2 per square metre, if the radius of the base is 14 metres.

**Solution.** Here, radius of the base \((r) = 14 \text{ cm}\) and height \((h)\) of the cylinder = 3 m.

CSA of the cylinder = \(2\pi rh\)
\[ = 2 \times \frac{22}{7} \times 14 \times 3 \text{ m}^2 \]
\[ = 264 \text{ m}^2 \]

Again, for the cone, we have \(r_1 = 14 \text{ m}, \ h_1 = 13.5 \text{ m} - 3 \text{ m} = 10.5 \text{ m}\)

and, slant height \((l_1) = \sqrt{r_1^2 + h_1^2} = \sqrt{14^2 + (10.5)^2} \text{ m} = \sqrt{196 + 110.25} \text{ m} \]
\[ = \sqrt{306.25} \text{ m} = 17.5 \text{ m} \]

\[ \therefore \text{CSA of the cone} = \pi r_1 l_1 = \frac{22}{7} \times 14 \times 17.5 \text{ m}^2 = 770 \text{ m}^2 \]

So, Total area which is to be painted = CSA of the cylinder + CSA of the cone
\[ = 264 \text{ m}^2 + 770 \text{ m}^2 = 1034 \text{ m}^2 \]

Hence, cost of painting = Rs. 1034 \times 2 = Rs. 2068 \text{ Ans.}
Example 6. A Gulab Jamun, contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 gulab jamuns, each shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm (see figure). [NCERT]

Solution.

For a cylindrical part, we have \( r = \frac{2.8}{2} \text{ cm} = 1.4 \text{ cm}, \ h = 5 \text{ cm} - 2.8 \text{ cm} = 2.2 \text{ cm} \)

Volume of one gulab jamun = volume of cylindrical part

\[ \pi r^2 h + 2 \times (\text{volume of hemispherical part}) \]

\[ = \pi r^2 h + 2 \times \frac{2}{3} \pi r^3 = \pi r^2 \left( h + \frac{4}{3}r \right) \]

\[ = \frac{22}{7} \times (1.4)^2 \times \left[ 2.2 + \frac{4}{3} \times (1.4) \right] \text{ cm}^3 \]

\[ = \frac{22}{7} \times 1.96 \times 7.2 \text{ cm}^3 \]

\[ \Rightarrow \text{Volume of such 45 gulab jamuns} = \frac{22}{7} \times 1.96 \times \frac{12.2}{3} \times 45 \text{ cm}^3 \]

Now, sugar syrup in such 45 gulab jamuns

\[ = 30\% \text{ of} \ \frac{22}{7} \times 1.96 \times \frac{12.2}{3} \times 45 \text{ cm}^3 \]

\[ = 338.0625 \text{ cm}^3 \approx 338 \text{ cm}^3 \text{ Ans.} \]

Example 7. A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4 cm. Find the volume of the wood in the entire stand. [NCERT]

Solution. For a cuboid, \( l = 15 \text{ cm}, \ b = 10 \text{ cm}, \ h = 3.5 \text{ cm} \)

\[ \therefore \text{Volume of cuboid} = l \times b \times h \]

\[ = 15 \times 10 \times \frac{35}{10} \text{ cm}^3 \]

For a conical depression,

\[ r = \frac{1}{2} \text{ cm}, \ h = 1.4 \text{ cm} \]
Volume of a conical part

\[
\frac{1}{3} \pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times \left( \frac{1}{2} \right)^2 \times 1.4 \text{ cm}^3
\]

\[
= \frac{11}{30} \text{ cm}^3
\]

Volume of wood used for pen stand

= Volume of cuboid – 4 \times (Volume of a conical part)

= 525 \text{ cm}^3 – 4 \times \left( \frac{11}{30} \right) \text{ cm}^3

= 525 \text{ cm}^3 – 1.47 \text{ cm}^3

= 523.53 \text{ cm}^3 \text{ Ans.}

Example 8. A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm.

NCERT]

Solution. For a conical part; \( r = 60 \text{ cm}, h = 120 \text{ cm} \).

For a hemisphere; \( r = 60 \text{ cm} \)

and, for a cylinder \( r = 60 \text{ cm}, H = 180 \text{ cm} \)

Now, volume of the solid consisting of a cone over a hemisphere

= Volume of cone + Volume of hemisphere

\[
= \frac{1}{3} \pi r^2 h + \frac{2}{3} \pi r^3 = \frac{1}{3} \pi r^2 (h + 2r)
\]

\[
= \frac{1}{3} \times \frac{22}{7} \times 60 \times 60 \times (120 + 2 \times 60) \text{ cm}^3
\]

\[
= \frac{1}{3} \times \frac{22}{7} \times 3600 \times 240 \text{ cm}^3 = \frac{6336000}{7} \text{ cm}^3
\]

Also, volume of cylindrical part = \( \pi r^2 H \)

\[
= \frac{22}{7} \times 60 \times 60 \times 180 \text{ cm}^3
\]

\[
= \frac{1425600}{7} \text{ cm}^3
\]

Volume of water left in the cylinder

= Volume of cylinder – Volume of given cone over a hemisphere

\[
= \frac{1425600}{7} \text{ cm}^3 - \frac{6336000}{7} \text{ cm}^3 = \frac{792000}{7} \text{ cm}^3
\]

= 1131428.571 cm³ = 1.131 m³ (approx) Ans.

Example 9. A vessel is in the form of a hemispherical bowl mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find its capacity.

\[
\text{Take } \pi = \frac{22}{7}
\]

[CBSE 2006 (C)]

Solution. For cylindrical part; \( r = 7 \text{ cm}, h = 6 \text{ cm} \)

and, for hemispherical part; \( r = 7 \text{ cm} \).
Total capacity of the bowl
= volume of the cylinder + volume of the hemisphere

\[ = \pi r^2 h + \frac{2}{3} \pi r^3 \]

\[ = \pi r^2 \left( h + \frac{2}{3} r \right) \]

\[ = \frac{22}{7} \times (7)^2 \left[ 6 + \frac{2}{3} \times (7) \right] \text{cm}^3 \]

\[ = 22 \times 7 \times \frac{32}{3} \text{cm}^3 \]

\[ = \frac{4928}{3} \text{cm}^3 = 1642.66 \text{ cm}^3 \ \text{Ans.} \]

**Example 10.** A sphere, of diameter 12 cm, is dropped in a right circular cylindrical vessel, partly filled with water. If the sphere is completely submerged in water, the water level in the cylindrical vessel rises by \(3 \frac{5}{9}\) cm. Find the diameter of the cylindrical vessel. [CBSE 2007]

**Solution.** Here, radius of a sphere \((r) = \frac{12}{2} \text{ cm} = 6 \text{ cm}\)

Volume of sphere \(= \frac{4}{3} \pi r^3 = \frac{4}{3} \pi (6)^3 \text{ cm}^3\)

Clearly, when a sphere is submerged into a cylindrical vessel, the volume of water displaced is equal to the volume of sphere.

Let radius of the cylindrical vessel be \(R \text{ cm}\). Then, volume of water displaced \(= \pi R^2 h\)

\[ = \pi R^2 \left( 3 \frac{5}{9} \right) \text{cm}^3 \]

according to question, \(\frac{4}{3} \pi (6)^3 = \pi R^2 \left( \frac{32}{9} \right)\)

\[ \Rightarrow R^2 = \frac{4 \times 6 \times 6 \times 6 \times 9}{3 \times 32} \Rightarrow R^2 = 81 \Rightarrow R = 9 \text{ cm} \]

\(\therefore\) diameter of the cylindrical vessel \(= 2R = 2 \times 9 \text{ cm} = 18 \text{ cm} \ \text{Ans.}\)

**Example 11.** A solid right circular cone of diameter 14 cm and height 8 cm is melted to form a hollow sphere. If the external diameter of the sphere is 10 cm, find the internal diameter of the sphere. [CBSE 2007]

**Solution.** For cone; radius \((r) = \frac{14}{2} \text{ cm} = 7 \text{ cm}\) and height \((h) = 8 \text{ cm}\)

Volume of a cone \(= \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi (7)^2 \times 8 \text{ cm}^3\)

For a hollow sphere; external radius \(= \frac{10}{2} \text{ cm} = 5 \text{ cm}\)

Let the internal radius be \(R \text{ cm}\).

then, volume of the hollow sphere \(= \frac{4}{3} \pi (5^3 - R^3)\)
Clearly, volume of cone = volume of hollow sphere
\[ \Rightarrow \quad \frac{1}{3} \pi (7)^2 \times 8 = \frac{1}{3} \pi (5^3 - R^3) \]
\[ \Rightarrow \quad 5^3 - R^3 = \frac{7 \times 7 \times 8}{4} \]
\[ \Rightarrow \quad R^3 = 27 \Rightarrow R^3 = 3^3 \Rightarrow R = 3 \text{ cm} \]
\[ \therefore \quad \text{internal diameter of the hollow sphere} = 2R = 2 \times 3 \text{ cm} = 6 \text{ cm Ans.} \]

**Example 12.** A well of diameter 3 m is dug 14 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width 4 m to form an embankment. Find the height of the embankment. [NCERT]

**Solution.**
Radius of the well \((r) = \frac{3}{2} \text{ m} = 1.5 \text{ m}\), and height \((h) = 14 \text{ m}\)

Volume of earth taken out of the well
\[ = \pi r^2 h = \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2} \times 14 \text{ m}^3 = 99 \text{ m}^3 \]

Outer radius of embankment \(R = \frac{3}{2} \text{ m} + 4 \text{ m} = \frac{11}{2} \text{ m}\)
\[ \therefore \quad \text{Area of embankment} = \text{outer area} - \text{inner area} \]
\[ = \pi R^2 - \pi r^2 \]
\[ = \pi (R^2 - r^2) \]
\[ = \pi \left[ \left( \frac{11}{2} \right)^2 - \left( \frac{3}{2} \right)^2 \right] \text{ m}^2 \]
\[ = \frac{22}{7} \times \left( \frac{11}{2} \times \frac{3}{2} \right) \times \left( \frac{11}{2} - \frac{3}{2} \right) \text{ m}^2 \]
\[ = \frac{22}{7} \times 7 \times 4 \text{ m}^2 = 88 \text{ m}^2 \]
\[ \therefore \quad \text{Height of embankment} = \frac{\text{Volume}}{\text{Area}} = \frac{99}{88} \text{ m} = \frac{9}{8} \text{ m} = 1.125 \text{ m Ans.} \]

**Example 13.** A container shaped like a right circular cylinder having diameter 12 cm and height 15 cm is full of ice cream. The ice cream is to be filled into cones of height 12 cm and diameter 6 cm, having a hemispherical shape on the top. Find the number of such cones which can be filled with ice cream. [NCERT, CBSE 2006(C)]

**Solution.**
For cylindrical container, we have \(R = \frac{12}{2} \text{ cm} = 6 \text{ cm}\) and \(H = 15 \text{ cm}\)

Volume of the cylindrical ice cream container = \(\pi R^2 H\)
\[ = \frac{22}{7} \times 6 \times 6 \times 15 \text{ cm}^3 \]
For a conical part of ice cream; \( r = \frac{6}{2} \text{ cm} = 3 \text{ cm} \), and \( h = 12 \text{ cm} \)

For a hemispherical part; \( r_1 = \frac{6}{2} \text{ cm} = 3 \text{ cm} \)

\[ \therefore \text{Volume of ice cream in the cone with a hemisphere over it} \]
\[ = \text{volume of cone} + \text{volume of hemisphere} \]
\[ = \frac{1}{3} \pi r^2 h + \frac{2}{3} \pi r_1^3 = \frac{1}{3} \pi r^2 [h + 2r] \quad (\because \ r = r_1) \]
\[ = \frac{1}{3} \times \frac{22}{7} \times (3)^2 \times [12 + 2 \times 3] \text{ cm}^3 \]
\[ = \frac{22}{7} \times 3 \times 18 \text{ cm}^3 \]

Let \( n \) be the number of such ice cream filled cones.

\[ \therefore \ n \times \text{volume of ice cream cones} = \text{volume of cylindrical ice cream container} \]
\[ \Rightarrow \ n \times \frac{22}{7} \times 3 \times 18 = \frac{22}{7} \times 6 \times 6 \times 15 \]
\[ \Rightarrow \ n = \frac{6 \times 6 \times 15}{3 \times 18} \Rightarrow n = 10 \]

\[ \therefore \ \text{Total number of such cones} = 10 \text{ Ans.} \]

**Example 14.** A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank in her field, which is 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of 3 km/hr, in how much time will the tank be filled? [NCERT]

**Solution.**

Speed of water = 3 km/hr = 3000 m/hr

Internal radius (\( r \)) of the pipe = \( \frac{20}{2} \text{ cm} = 10 \text{ cm} = \frac{1}{10} \text{ m} \)

\[ \therefore \ \text{Volume of water that flows out in 1 hour} \]
\[ = \pi r^2 h = \pi \times \left( \frac{1}{10} \right)^2 \times 3000 \text{ m}^3 = 30 \pi \text{ m}^3 \]

Let the time taken to fill the tank be \( t \) hours.

\[ \therefore \ \text{Volume of the water that flows out in} \ t \ \text{hours} = 30 \pi \ t \text{ m}^3 \]

Radius of the base of the tank (\( R \)) = \( \frac{10}{2} \text{ m} = 5 \text{ m} \) and depth (\( H \)) of the tank = 2 m

\[ \therefore \ \text{Volume of the cylindrical tank} = \pi R^2 H \]
\[ = \pi (5)^2 \times 2 \text{ m}^3 = 50 \pi \text{ m}^3 \]

According to given question, volume of water that flows out in \( t \) hours = volume of the tank
\[ \Rightarrow \ 30 \pi t = 50 \pi \]
\[ \Rightarrow \ t = \frac{50}{30} \text{ hours} = \frac{5}{3} \times 60 \text{ minutes} = 100 \text{ minutes} \text{ Ans.} \]
Example 15. A container, opened from the top and made up of a metal sheet, is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm, respectively. Find the cost of the milk which can be completely fill the container, at the rate of Rs. 20 per litre. Also find the cost of metal sheet used to make the container, if it costs Rs. 8 per 100 cm$^2$ [Take $\pi = 3.14$]

Solution. Here, $h = 16$ cm, $r = 8$ cm, $R = 20$ cm

Slant height $l = \sqrt{R^2 + (R-r)^2} = \sqrt{(16)^2 + (20-8)^2}$

= $\sqrt{256 + 144} = \sqrt{400} = 20$ cm

Now, volume of frustum $= \frac{1}{3} \pi h (r^2 + R^2 + rR)$

= $\frac{1}{3} \times 3.14 \times 16 \times [(8)^2 + (20)^2 + (8)(20)] \text{ cm}^3$

= $\frac{1}{3} \times 3.14 \times 16 \times (64 + 400 + 160) \text{ cm}^3$

= $\frac{1}{3} \times 3.14 \times 16 \times 624 \text{ cm}^3 = 10449.92 \text{ cm}^3 = 10.4492 \text{ litre}$ (\because 1 l = 1000 cm$^3$)

\[ \therefore \text{ cost of milk } = \text{ Rs. } 20 \times 10.45 = \text{ Rs. } 209 \]

Now, surface area of the container $= \pi l (r + R) + \pi r^2$

= $3.14 \times 20 \times (8 + 20) \text{ cm}^2 + 3.14 \times (8)^2 \text{ cm}^2$

= 1758.4 cm$^2 + 200.96 \text{ cm}^2$

= 1959.36 cm$^2$

Now, cost of metal sheet = Rs. 8 per 100 cm$^2$

\[ \therefore \text{ cost of metal sheet required } = \text{ Rs. } 159.36 \times \frac{8}{100} = \text{ Rs. } 156.75 \] (approx) Ans.

Example 16. A metallic right circular cone height 20 cm and whose vertical angle is $60^\circ$ is cut into two parts at the middle of its height by a plane parallel to its base. If the frustum so obtained be drawn into a wire of diameter $\frac{1}{16}$ cm, find the length of the wire. [NCERT]

Solution. Clearly, $OE = \frac{1}{2} OF = \frac{1}{2} \times 20 \text{ cm} = 10$ cm

Now, in right angled $\triangle OEC$, $\frac{CE}{OE} = \tan 30^\circ \Rightarrow CE = OE \tan 30^\circ$

$\Rightarrow CE = 10 \times \frac{1}{\sqrt{3}} \text{ cm} = \frac{10}{\sqrt{3}} \text{ cm}$

Also, $\triangle OCE \sim \triangle OAF$ (AA similarity)

$\Rightarrow \frac{OE}{OF} = \frac{CE}{AF} \Rightarrow \frac{10}{20} = \frac{10}{\sqrt{3}} \times \frac{1}{AF}$

$\Rightarrow AF = \frac{20}{\sqrt{3}} \text{ cm}$
Now, for the frustum CDBA, we have

\[ r = CE = \frac{10}{\sqrt{3}} \text{ cm}, \quad R = AF = \frac{20}{\sqrt{3}} \text{ cm} \]

and, \( h = 20 \text{ cm} - 10 \text{ cm} = 10 \text{ cm} \)

\[ \therefore \text{Volume of frustum CDBA} = \frac{1}{3} \pi h (r^2 + R^2 + rR) \]

\[ = \frac{\pi}{3} \times 10 \left[ \left( \frac{10}{\sqrt{3}} \right)^2 + \left( \frac{20}{\sqrt{3}} \right)^2 + \left( \frac{10}{\sqrt{3}} \right) \left( \frac{20}{\sqrt{3}} \right) \right] \text{ cm}^3 \]

\[ = \frac{10\pi}{3} \left[ \frac{100}{3} + \frac{400}{3} + \frac{200}{3} \right] \text{ cm}^3 \]

\[ = \frac{10\pi}{3} \left( \frac{700}{3} \right) \text{ cm}^3 = \frac{7000}{9} \pi \text{ cm}^3 \]

Also, volume of wire formed \( = \pi R^2 l \)

\[ = \pi \times \left( \frac{1}{2 \times 16} \right)^2 \times l \text{ cm}^3 \]

According to given question,

Volume of material used for cylindrical wire \( = \) volume of frustum CDBA.

\[ \Rightarrow \pi \times \left( \frac{1}{2 \times 16} \right)^2 \times l = \frac{7000}{9} \pi \]

\[ \Rightarrow l = \frac{2 \times 16 \times 16 \times 7000}{9} \text{ cm} = \frac{7168000}{9} \text{ cm} \]

\[ = 796444.4 \text{ cm} \]

\[ = 7964.44 \text{ m} \quad \text{Ans.} \]

**PRACTICE EXERCISE**

**Questions based on surface area and volume of combination of solids**

1. Three cubes each of side 5 cm are joined end to end. Find the surface area of the resulting cuboid.
2. A circus tent is cylindrical up to a height of 3 m and conical above it. If the diameter of the base is 105 m and the slant height of the conical part is 53 m, find the total canvas used in making the tent.

   \[ \text{[CBSE 2004]} \]

3. A solid wooden toy is in the shape of a right circular cone mounted on a hemisphere. If the radius of the hemisphere is 4.2 cm and the total height of the toy is 10.2 cm, find the volume of the wooden toy.
4. A solid is in the form of a cylinder with hemispherical ends. The total height of the solid is 19 cm and the diameter of the cylinder is 7 cm. Find the volume and total surface area of the solid. \[ \text{[use } \pi = \frac{22}{7} \text{]} \]
5. A solid is composed of a cylinder with hemispherical ends. If the whole length of the solid is 108 cm and the diameter of the hemispherical ends is 36 cm, find the cost of polishing the surface of the solid at the rate of 10 paisa per sq. cm. \[ \text{use } \pi = \frac{22}{7} \] 

6. A solid toy is in the form of a right circular cylinder with a hemispherical shape at one end and a cone at the other end. Their common diameter is 4.2 cm and the height of the cylindrical and conical portions are 12 cm and 7 cm respectively. Find the volume of the solid toy. \[ \text{use } \pi = \frac{22}{7} \] [CBSE 2002(C)]

7. A solid is composed of a cylinder with hemispherical ends. If the whole length of the solid is 104 cm and the radius of each of the hemispherical ends is 7 cm, find the cost of polishing its surface at the rate of Rs. 10 per dm². [CBSE 2006(C)]

8. A cylindrical road roller made of iron is 1 m long. Its internal diameter is 54 cm and the thickness of the iron sheet used in making the roller is 9 cm. Find the mass of the roller, if 1 cm³ of iron has 7.8 gm mass. [use \( \pi = 3.14 \)]

9. A tent is in the form of a cylinder of diameter 20 m and height 2.5 m, surmounted by a cone of equal base and height 7.5 m. Find the capacity of the tent and the cost of the canvas at Rs. 100 per square meter.

10. A cylindrical tub of radius 5 cm and length 9.8 cm is full of water. A solid in the form of a right circular cone mounted on a hemisphere is 3.5 cm and height of the cone outside the hemisphere is 5 cm, find the volume of the water left in the tub. \[ \text{use } \pi = \frac{22}{7} \] [CBSE 2000 (C)]

11. A toy is in the shape of a right circular cylinder with a hemisphere on one end and a cone on the other. The radius and height of the cylindrical part are 5 cm and 13 cm respectively. The radii of the hemispherical and conical parts are the same as that of the cylindrical parts. Find the surface area of the toy if the total height of the toy is 30 cm. [CBSE 2002]

12. A vessel is in the form of inverted cone. Its height is 8 cm and radius of its top, which is open, is 5 cm. It is filled with water up to the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped in the vessel, one fourth of the water flows out. Find the number of lead shots dropped in the vessel. [NCERT]

13. A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by a cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm³ of iron has approximately 8 g mass. [use \( \pi = 3.14 \)] [NCERT]

14. A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also find the cost of the canvas of the tent at the rate of Rs. 500 per m². [NCERT]

15. A cylindrical container of radius 6 cm and height 15 cm is filled with ice-cream. The whole ice-cream has to be distributed to 10 children in equal cones with hemispherical tops. If the height of the conical portion is four times the radius of its base, find the radius of the ice-cream cone.
16. A solid is in the form of a right circular cone mounted on a hemisphere. The radius of the hemisphere is 3.5 cm and the height of the cone is 4 cm. The solid is placed in a cylindrical tub, full of water, in such a way that the whole solid is submerged in water. If the radius of the cylinder is 5 cm and its height is 10.5 cm, find the volume of water left in the cylindrical tub. \[ \text{use } \pi = \frac{22}{7} \]

17. The interior of a building is in the form of a right circular cylinder of diameter 4.2 m and height 4 m, surmounted by a cone. The vertical height of the cone is 2.1 m. Find the outer surface area and the volume of the building. \[ \text{use } \pi = \frac{22}{7} \]

18. A building is in the form of a cylinder surmounted by a hemispherical vaulted dome and contains \( \frac{41}{21} \text{ cm}^3 \) of air. If the internal diameter of the building is equal to the total height above the floor, find the height of the building.

19. An ice-cream cone consists of a right circular cone of height 14 cm and diameter of the circular top is 5 cm. It has hemisphere on the top with the same diameter as of circular top. Find the volume of ice-cream in the cone.

20. A circus tent is in the form of a right circular cylinder and a right circular cone above it. The diameter and the height of the cylindrical part of the tent are 126 m and 5 m respectively. The total height of the tent is 21 m. Find the total surface area of the tent. \[ \text{use } \pi = \frac{22}{7} \]

21. An iron pillar has some part in the form of a right circular cylinder and the remaining in the form of a right circular cone. The radius of the base of each of the cone and the cylinder is 8 cm, the cylindrical part is 240 cm high and conical part is 36 cm high. Find the weight of the pillar, if 1 cm\(^3\) of iron weighs 10 grams. \[ \text{use } \pi = \frac{22}{7} \]

22. The height of a solid cylinder is 15 cm and diameter 7 cm. Two equal conical holes of radius 3 cm and height 4 cm are cut off. Find the volume and surface area of the remaining solid.

![Diagram](image.png)

23. A right triangle whose sides are 15 cm and 20 cm, is made to revolve above its hypotenuse. Find the volume and the surface area of the double cone so formed. (use \( \pi = 3.14 \)).

24. Find the volume and the surface area of a solid in the form of a right circular cylinder with hemispherical ends whose total length is 2.7 m and the diameter of each hemispherical ends is 0.7 m.
25. A wooden toy rocket is in the shape of a cone mounted on a cylinder, as shown in the given figure. The height of the entire rocket is 26 cm, while the height of the conical part is 6 cm. The base of the conical portion has a diameter of 5 cm, while the base diameter of the cylindrical portion is 3 cm. If the conical portion is to be painted orange and the cylindrical portion yellow, find the area of the rocket painted with each of these colours. (use $\pi = 3.14$)

Question based on conversion of solid from one shape to another

26. Find the volume of the largest right circular cone that can be cut out of a cube whose edge is 9 cm.
27. The largest sphere is carved out of a cube of a side 7 cm. Find the volume of the sphere.
28. Determine the ratio of the volume of a cube to that of a sphere which will exactly fit inside the cube.
29. A solid sphere of radius 3 cm is melted and then recast into small spherical balls each of diameter 0.6 cm. Find the number of balls thus obtained.
30. A spherical ball of radius 3 cm is melted and recast into three spherical balls. The radii of the two of the balls are 1.5 cm and 2 cm respectively. Determine the radius of the third ball.
31. How many spherical lead shots each 4.2 cm in diameter can be obtained from a rectangular solid of lead with dimensions 66 cm $\times$ 42 cm $\times$ 21 cm. [use $\pi = \frac{22}{7}$]
32. A hemispherical bowl of internal diameter 36 cm contains a liquid. This liquid is to be filled in cylindrical bottles of radius 3 cm and height 6 cm. How many bottles are required to empty the bowl?
33. A conical vessel whose internal radius is 5 cm and height 24 cm is full of water. The water is emptied into a cylindrical vessel with internal radius 10 cm. Find the height to which the water rises.
34. The radii of the internal and external surfaces of a metallic spherical shell are 3 cm and 5 cm respectively. It is melted and recast into a solid right circular cylinder of height $10\frac{2}{3}$ cm. Find the radius of the base of the cylinder.
35. A sphere of diameter 6 cm is dropped in a right circular cylindrical vessel partly filled with water. The diameter of the cylindrical vessel is 12 cm. If the sphere is completely submerged in water, by how much will the level of water rise in the cylindrical vessel?
36. A glass cylinder with diameter 20 cm has water to the height of 9 cm. A metal cube of 8 cm edge is immersed in it completely. Find the height by which the water will rise in the cylinder. (use $\pi = 3.142$)
37. A cylindrical jar is 25 cm high with internal diameter 7 cm. A metal cube of edge 4 cm is immersed in the jar. Find the rise in the level of water. \[
\text{use } \pi = \frac{22}{7}
\]

38. An iron sphere of diameter 12 cm is dropped into a cylindrical can of diameter 24 cm containing water. Find the rise in the level of water when the sphere is completely immersed.

39. A solid sphere of radius 6 cm is melted and recast into a hollow cylinder of uniform thickness. If the external radius of the base of the cylinder is 4 cm and if its height is 76.8 cm, find the uniform thickness of the cylinder.

40. A cylindrical can of internal diameter 21 cm contains water. A solid sphere whose diameter is 10.5 cm is lowered into the cylindrical can. The sphere is completely immersed in water. Find the rise in the water level, assuming that no water overflows.

41. A cylindrical bucket 42 cm in diameter and 50 cm high is full of water. The water is emptied into a rectangular tank 66 cm long and 21 cm wide. Find the height of the water level in the tank.

42. A rectangular vessel is 20 cm × 16 cm × 11 cm and is full of water. This water is poured into a conical vessel of base radius 10 cm. If the vessel is completely filled, find the height of the conical vessel.

43. A spherical copper ball of a diameter 9 cm is melted and drawn into a wire of diameter 2 cm. Find the length of the wire in metres.

44. Lead spheres of diameter 6 cm are dropped into a beaker containing some water and are fully submerged. The diameter of the beaker is 18 cm. Find how many lead spheres have been dropped in if the water rises by 40 cm?

45. A hemispherical bowl of internal radius 9 cm is full of liquid. This liquid is to be filled into cylindrical shaped small bottles each of diameter 3 cm and height 4 cm. How many bottles are necessary to empty the bowl? [CBSE 2005]

46. A cylindrical tub of radius 12 cm contains water to a depth of 20 cm. A spherical ball is dropped into the tub and the level of the water is raised by 6.75 cm. Find the diameter of the ball.

47. The diameters of the internal and external surfaces of a hollow spherical shell are 6 cm and 10 cm respectively. If it is melted and recast into a solid cylinder of diameter 14 cm, find the height of the cylinder. [CBSE 2001(C)]

48. Four right circular cylindrical vessels each having diameter 21 cm and height 38 cm are full of ice-cream. The ice-cream is to be filled in cones of height 12 cm and diameter 7 cm having a hemispherical shape at the top. Find the total number of such cones which can be filled with ice-cream. [CBSE 2002]

49. A well, whose diameter is 7 m, has been dug 22.5 m deep and the earth dug out is used to form an embankment 10.5 m wide around it. Find the height of the embankment.

50. An agricultural field is in the form of a rectangle of length 20 m and width 14 m. A 10 m deep well of diameter 7 m is dug in a corner of the field and the earth taken out of the well is spread evenly over the remaining part of the field. Find the rise in its level.

51. A solid cylinder of diameter 12 cm and height 15 cm is melted and recast into 12 toys in the shape of a right circular cone mounted on a hemisphere. Find the radius of the hemisphere and the total height of the toy if height of the cone is three times the radius.

52. A metallic sphere of radius 10.5 cm is melted and then recast into small cones, each of radius 3.5 cm and height 3 cm. Find how many cones are obtained? [CBSE 2002]

53. The base radius and height of a right circular solid cone are 2 cm and 8 cm respectively. It is melted and recast into spheres of diameter 2 cm each. Find the number of spheres so formed. [CBSE 2005]
54. A conical vessel of radius 6 cm and height 8 cm is completely filled with the water. A sphere is lowered into the water and its size is such that when it touches the sides, it is just immersed as shown in the given figure. What fraction of the water overflows?

55. Water is flowing at the rate of 0.7 metres per second through a circular pipe whose internal diameter is 2 cm into a cylindrical tank, the radius of whose base is 40 cm. Determine the increase in the water level in $\frac{1}{2}$ hour. [CBSE 2006(C)]

56. Water in a canal, 30 dm wide and 12 dm deep is flowing with velocity of 10 km/hr. How much area will it irrigate in $\frac{1}{2}$ hour, if 8 cm of standing water is required for irrigation?

57. Water is being pumped out through a circular pipe whose internal diameter is 7 cm. If the flow of water is 72 cm per second, how many litres of water are being pumped out in one hour?

58. A hemispherical tank of radius $1\frac{3}{4}$ m is full of water. It is connected by a cylindrical pipe which empties it at 7 litres per second. Find the time it will take to empty the tank completely.

59. Water flows at the rate of 10 metres per minute from a cylindrical pipe 5 mm in diameter. How long would it take to fill a conical vessel whose diameter at the surface is 40 cm and depth 24 cm?

60. Water is flowing at the rate of 2.5 km/hr through a circular pipe of 20 cm internal diameter into a circular cistern of diameter 20 metres and depth 2.5 metres. In how much time will the cistern be filled?

Questions based on frustum of a cone

61. The radii of the circular ends of a frustum of height 6 cm are 14 cm and 6 cm respectively. Find the lateral surface area and total surface area of the frustum.

62. If the radii of the circular ends of a conical bucket which is 45 cm high be 28 cm and 7 cm, find the capacity of the bucket. [CBSE 2004,05]

63. The radii of the circular ends of a solid frustum of a cone are 33 cm and 27 cm and its slant height is 10 cm. Find its total surface area. [CBSE 2005]

64. A friction clutch is in the form of frustum of a cone, the diameters of the ends being 32 cm and 20 cm and length 8 cm. Find its bearing surface and volume.

65. A bucket made of aluminium sheet is of height 20 cm and its upper and lower ends are of radius 25 cm and 10 cm respectively. Find the cost of making the bucket, if the aluminium sheet costs Rs. 70 per 100 cm². [use $\pi = 3.14$] [CBSE 2006(C)]
66. A bucket is in the form of a frustum of a cone with a capacity of 12308.8 cm$^3$ of water. The radii of the top and bottom circular ends are 20 cm and 12 cm respectively. Find the height of the bucket and the area of the metal sheet used in its making. [use $\pi = 3.14$] [CBSE 2006 (C)]

67. The radii of the ends of a bucket 30 cm high are 21 cm and 7 cm. Find its capacity in litres and the amount of sheet required to make this bucket and its cost at Rs. 2 per sq. dm of sheet.

68. A milk container is made of metal sheet in the form of a frustum of a cone and is of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm respectively. Find the cost of milk which the container can hold when fully filled at Rs 16 per litre and, the cost of the metal sheet used in making the container, at Rs 5 per sq. dm. [use $\pi = 3.14$]

69. A shuttlecock used for playing badminton has the shape of a frustum of a cone mounted on a hemisphere. The external diameters of the frustum are 5 cm and 2 cm, and the height of the entire shuttlecock is 7 cm. Find its external surface area.

70. A right circular cone is divided by a plane parallel to its base in two equal volumes. In what ratio will the plane divide the axis of the cone?

71. The height of a cone is 30 cm. A small cone is cut off at the top by a plane parallel to the base. If its volume be $\frac{1}{27}$ of the volume of the given cone, at what height above the base is the section made? [CBSE 2005 (C)]

72. A hollow cone is cut by a plane parallel to the base and the upper partion is removed. If the curved surface of the remainder is $\frac{8}{9}$ of the curved surface of the whole cone, find the ratio of the line-segment into which the cones altitude is divided by the plane. [CBSE 2004, 2004 (C)]

73. If a cone of radius 10 cm is divided into two parts by drawing a plane through the mid-point of its axis, parallel to its base. Compare the volumes of the two parts. [CBSE 2000 (C)]

74. A bucket of height 8 cm and made up of copper sheet is in the form of a frustum of a right circular cone with radii of its lower and upper ends as 3 cm and 9 cm respectively. Calculate:
   (i) the height of the cone of which the bucket is a part.
   (ii) the volume of water which can be filled in the bucket.
   (iii) the area of copper sheet required to make the bucket.
   (Leave the answer in terms of $\pi$) [CBSE sample paper 2004]

75. The height of a right circular cone is trisected by two planes parallel to the base at equal distances. Show that the volumes of the three portions, starting from the top, are in the ratio of 1 : 7 : 19.

**Hints to Selected Questions**

5. Surface Area of solid = S.A. of cylinder + 2 (S.A. of hemi-sphere)

\[ = 2\pi rh + 2(2\pi r^2) \]

\[ = 2\pi r(h + 2r) = \frac{22}{7} \times 18 \times [72 + 2(18)] \text{ cm}^2 \]

\[ = 12219.4 \text{ cm}^2 \]

\[ \therefore \text{ Required cost} = \text{Rs. 12219.4} \times \frac{10}{100} = \text{Rs. 1221.94} \]
8. Volume of road roller
\[ V = \pi (R^2 - r^2)h \]
\[ = \pi (36^2 - 27^2) \times 100 \text{ cm}^2 = 178038 \text{ cm}^2 \]
\[ \therefore \text{Mass} = 178038 \times 7.8 \text{ gm} = 1388.7 \text{ kg} \]

12. Let number of lead shots be \( n \). Then,
\[ n \times \text{volume of 1 lead shot} = \frac{1}{4} \text{(volume of water that flows out)} \]
\[ \Rightarrow n \times \frac{4}{3} \pi (0.5)^3 = \frac{1}{4} \left[ \frac{1}{3} \pi (5)^2 \times 8 \right] \]
\[ \Rightarrow n = 100 \]

15. Let radius of base of ice-cream cone be \( r \) cm. Then, its height = 4\( r \)
According to question,
\[ \pi (6)^2 \times 15 = 10 \times \left[ \frac{1}{3} \pi (r)^2 (4r) + \frac{2}{3} \pi r^3 \right] \]
\[ \Rightarrow r = 3 \text{ cm}. \]

23. Here, \( BC = \sqrt{AB^2 + AC^2} = \sqrt{15^2 + 20^2} \text{ cm} = 25 \text{ cm} \)
Let \( OA = y \) and \( OB = x \).
In \( \Delta OAB \) and \( \Delta OAC \), \( x^2 + y^2 = 15^2 \) and \( (25 - x)^2 + y^2 = 20^2 \)
solving together, we get \( x = 9 \) and \( y = 12 \).
Thus, \( OA = 12 \text{ cm} \) and \( OB = 9 \text{ cm} \).

Now, Volume of double cone
\[ = \frac{1}{3} \pi (OA)^2 \times OC + \frac{1}{3} \pi (OA)^2 \times OB \]
\[ = \frac{1}{3} \pi (OA)^2 \times (OC + OB) \]
\[ = \frac{1}{3} \times 3.14 \times (12)^2 \times 25 \text{ cm}^3 = 3768 \text{ cm}^3 \]
Also, surface area of double cone \[ = \pi \times OA \times AC + \pi \times OA \times AB \]
\[ = \pi \times 12 \times 20 \text{ cm}^2 + \pi \times 12 \times 15 \text{ cm}^2 \]
\[ = 420\pi \text{ cm}^2 = 1318.8 \text{ cm}^2 \]

25. For cylindrical portion; \( r = \frac{3}{2} \text{ cm}, h = (26 - 6) \text{ cm} = 20 \text{ cm} \)

For conical portion; \( R = \frac{5}{2} \text{ cm}, h = 6 \text{ cm}. \)

Now, slant height \( (l) = \sqrt{h^2 + R^2} = \sqrt{(6)^2 + \left(\frac{5}{2}\right)^2} \text{ cm} = \frac{13}{2} \text{ cm}. \)

The area to be painted orange
\[ = \text{CSA of cone + Base area of cone – Base area of cylinder} \]
\[ = \pi RL + \pi R^2 - \pi r^2 = \pi (RL + R^2 - r^2) \]
\[ = 3.14 \left[ \frac{5}{2} \times \frac{13}{2} + \left(\frac{5}{2}\right)^2 - \left(\frac{3}{2}\right)^2 \right] \text{ cm}^2 = 63.585 \text{ cm}^2 \]

Also, area to be painted yellow
\[ = \text{CSA of cylinder + Area of the one base of the cylinder} \]
\[ = 2\pi rh + \pi r^2. \]
Substitute the values and solve.

34. Here, volume of spherical shell = volume of cylinder.
\[ \Rightarrow \frac{4}{3}\pi (5^3 - 3^3) = \pi r^3\left(\frac{32}{3}\right) \Rightarrow r = 3.5 \text{ cm} \]

35. Let the level of water rise in cylindrical vessel be \( r \text{ cm}. \)

Then, \( \frac{4}{3}\pi (3^3) = \pi (6)^2 \times h \Rightarrow h = 1 \text{ cm} \)

54. In \( \triangle VO'A, VA = \sqrt{8^2 + 6^2} \text{ cm} = 10 \text{ cm} \)

Now, \( AO' = AP \Rightarrow AP = 6 \text{ cm} \)
\[ \therefore VP = VA - AP = 4 \text{ cm} \]

Now, \( VO = VO' - OO' = (8 - r) \text{ cm} \)

In \( \triangle VPO, VO^2 = VP^2 + OP^2 \)
\[ \Rightarrow (8 - r)^2 = 16 + r^2 \Rightarrow r = 3 \text{ cm}. \]

Now, volume of the sphere \( (V_1) = \frac{4}{3}\pi (3)^3 \text{ cm}^3 = 36 \pi \text{ cm}^3 \)

and, volume of water \( (V_2) = \text{volume of cone} = \frac{1}{3}\pi \times (6)^2 \times 8 \text{ cm}^3 = 96 \pi \text{ cm}^3 \)

\[ \therefore \text{fraction of the water that flows out} = \frac{V_1}{V_2} = \frac{3}{8}. \]

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55. Length of water that flows out in 30 minutes
   \[ (0.7 \times 100 \times 60 \times 30) \text{ cm} = 126000 \text{ cm}. \]
   Volume of water that flows out in 30 minutes.
   \[ \pi r^2 h = \pi \times (1)^2 \times 126000 \text{ cm}^3 = 126000 \pi \text{ cm}^3 \]
   Let the depth of water in the tank be \( x \) cm.
   Volume of the water in the tank \( \pi R^2 H = \pi \times (40)^2 \times x \text{ cm}^3 \)
   According to question, \( \pi \times (40)^2 \times x = 126000 \pi \Rightarrow x = 78.75 \text{ cm} \)

71. Volume of given cone \( \frac{1}{3} \pi R^2 \times 30 \text{ cm}^3 = 10 \pi R^2 \text{ cm}^3 \).
   Now, volume of smaller cone \( \frac{1}{3} \pi r^2 h \text{ cm}^3 \)
   according to question, \( \frac{1}{3} \pi r^3 h = \frac{1}{27} (10 \pi R^2) \)
   \[ \Rightarrow \left( \frac{R}{r} \right)^2 = \frac{9h}{10} \] ...(1)
   Now, \( \triangle OAB \sim \triangle OCD \Rightarrow \frac{AB}{CD} = \frac{OA}{OC} \)
   \[ \Rightarrow \frac{R}{r} = \frac{30}{h} \] ...(2)
   from (1) and (2), we get \( \left( \frac{30}{h} \right)^2 = \frac{9h}{10} \Rightarrow h^3 = 1000 \Rightarrow h = 10 \)
   \[ \therefore \] Height of the section from the base = 30 cm – 10 cm = 20 cm.

72. Clearly, \( \triangle OED \sim \triangle OFB \)
   \[ \Rightarrow \frac{h}{H} = \frac{l}{L} = \frac{r}{R} \] ...(1)
   Now, CSA of frustum CABLED = \( \frac{8}{9} \) (CSA of cone OAB)
   \[ \Rightarrow \text{CSA of cone OCD} = \frac{1}{9} \) (CSA of cone OAB)
   \[ \Rightarrow \pi rl = \frac{1}{9} \pi RL \Rightarrow \left( \frac{r}{R} \right) \left( \frac{l}{L} \right) = \frac{1}{9} \]
   \[ \Rightarrow h \]
   \[ \Rightarrow H = 3h. \]
   Now, \( EF = OF - OE = H - h = 3h - h = 2h. \)
   \[ \therefore \frac{OE}{EF} = \frac{h}{2h} = \frac{1}{2}. \]
75. Clearly, \( \Delta VOA \sim \Delta VOA' \)

\[
\frac{VO}{VO'} = \frac{OA}{O'A'} \Rightarrow \frac{r}{r_i} = \frac{3h}{2h} \Rightarrow r_i = \frac{2r}{3}
\]

Also, \( \Delta VOA \sim \Delta VLC \)

\[
\frac{VO}{VL} = \frac{OA}{LC} \Rightarrow \frac{3h}{h} = \frac{r_i}{r_2} \Rightarrow r_2 = \frac{r}{3}
\]

Now, Volume \( (V_1) \) of cone VCD = \( \frac{1}{3} \pi r_i^2 h \)

\[
= \frac{1}{3} \pi \left( \frac{r}{3} \right)^2 h = \frac{1}{27} \pi r^2 h
\]

Volume \( (V_2) \) of frustum A'B'DC = \( \frac{1}{3} \pi (r_i^2 + r_2^2 + r_3r_2) h \)

\[
= \frac{1}{3} \pi \left( \frac{4r^2}{9} + \frac{r^2}{9} + \frac{2r^2}{9} \right) h = \frac{7}{27} \pi r^2 h
\]

Volume \( (V_3) \) of frustum ABB'A' = \( \frac{1}{3} \pi (r^2 + r_i^2 + r_3r_i) h \)

\[
= \frac{19}{27} \pi r^2 h
\]

\[
\therefore \text{ Required ratio } = V_1 : V_2 : V_3 = 1:7:19.
\]

**MULTIPLE CHOICE QUESTIONS**

**Mark the correct alternative in each of the following :**

1. The height of a right circular cone is 12 cm. If its volume is \( 100\pi \text{ cm}^3 \), its slant height is:
   
   (a) 18 cm \hspace{1cm} (b) 20 cm \hspace{1cm} (c) 13 cm \hspace{1cm} (d) none of these

2. The volume of a largest sphere which is carved out of a cube of side 21 cm is:
   
   (a) 3951 \text{ cm}^3 \hspace{1cm} (b) 4851 \text{ cm}^3 \hspace{1cm} (c) 5351 \text{ cm}^3 \hspace{1cm} (d) none of these

3. A solid cylinder of brass 8 m high and 4 m in diameter is melted and recast into a cone of diameter 3 m. The height of the cone is:
   
   (a) \( 42\frac{2}{3} \) m \hspace{1cm} (b) 42 m \hspace{1cm} (c) \( 45\frac{1}{3} \) m \hspace{1cm} (d) none of these

4. A cone is 8.4 cm high and radius of its base is 2.1 cm. It is melted and recast into a sphere. The radius of the sphere is:
   
   (a) 2 cm \hspace{1cm} (b) 2.1 cm \hspace{1cm} (c) 3.5 cm \hspace{1cm} (d) none of these

5. The diameter of a metallic sphere is 6 cm. The sphere is melted and drawn into a wire of uniform circular cross-section. If the length of the wire is 36 m, its radius is:
   
   (a) 0.5 mm \hspace{1cm} (b) 1 mm \hspace{1cm} (c) 2 mm \hspace{1cm} (d) 4 mm
6. A spherical object of radius 14 cm is dropped into water contained in a right circular cylindrical vessel of radius 21 cm. If the object is completely immersed, the level of water is raised by:
   (a) 6.5 cm  (b) 7.3 cm  (c) 8.3 cm  (d) none of these

7. Marbles of diameter 1.4 cm are dropped into a cylindrical beaker, of diameter 7 cm, containing some water. The number of marbles that should be dropped into the beaker so that the water level rises by 5.6 cm, is:
   (a) 50  (b) 100  (c) 150  (d) 200

8. A cone and a hemisphere have equal bases and equal volumes. The ratio of their heights is:
   (a) 1 : 2  (b) 2 : 1  (c) 1 : 3  (d) 3 : 1

9. A cylinder whose height is two-third of its diameter has the same volume as a sphere of radius 4 cm. The radius of the base of the cylinder is:
   (a) 2 cm  (b) 3 cm  (c) 4 cm  (d) 6 cm

10. The diameter of a copper sphere is 18 cm. The sphere is melted and is drawn into a long wire of uniform circular cross-section. If the length of the wire is 108 m, its diameter is:
    (a) 0.3 cm  (b) 0.6 cm  (c) 0.8 cm  (d) none of these

11. A circus tent is cylindrical to a height of 4 m and conical above it, if its diameter is 105 m and its slant height is 40 m, the total area of the canvas required in $m^2$ is:
    (a) 2560  (b) 3760  (c) 7920  (d) none of these

12. A hollow spherical shell is made of density $\frac{4}{9} g/cm^3$. If its internal and external radii are 8 cm and 9 cm respectively, the weight of the shell is: [use $\pi = 3.141$]
    (a) 203.91 g  (b) 303.91 g  (c) 403.91 g  (d) 503.91 g

13. The rainfall from a roof 22 m × 20 m drains into a conical vessel having a diameter of base as 2m and height 3.5 m. If the vessel is just full, the rainfall is:
    (a) $\frac{4}{5}$ cm  (b) $\frac{5}{6}$ cm  (c) $\frac{6}{7}$ cm  (d) none of these

14. The number of spherical bullets each of diameter 2 cm which can be made out of a cube of lead whose edge measures 22 cm is
    (a) 1550  (b) 2050  (c) 2241  (d) 2541

15. A reservoir is in the shape of a frustum of a right circular cone. It is 8 m across at the top and 4m across at the bottom. If it is 6 m deep, then its capacity is:
    (a) 176 m$^3$  (b) 186 m$^3$  (c) 216 m$^3$  (d) 254 m$^3$

**VERY SHORT ANSWER TYPE QUESTIONS (1 MARK QUESTIONS)**

1. What is the surface area of a cube, whose volume is 27 cm$^3$?
2. What is the length of a diagonal of a cube that can be inscribed in a sphere of radius 6 cm?
3. A rectangular sheet of paper 66 cm × 18 cm is rolled along its length and a cylinder is formed. What is curved surface area of the cylinder.
4. The largest sphere is carved out of a cube of side 7 cm. What is the volume of the sphere in terms of $\pi$.
5. The radii and heights of a cone, a hemisphere and cylinder are same. What is the ratio of the volumes of them.
6. What is the height of a cone whose base area and volume are numerically equal?
7. What is the length of the diagonal of a cube that can be inscribed in a sphere of radius 10 cm?
8. Three cubes, whose edges are 6 cm, 8 cm and 10 cm are melted and formed into a single cube. What is the length of diagonal of the single cube?
9. If the volume and surface area of a sphere are numerically equal, then what is its diameter?
10. What is the length of the largest rod that can be put in a box of inner dimensions 18 cm × 24 cm × 30 cm?
11. What is the slant height of a cone, if its height is 12 cm and radius is 5 cm?
12. If the semi-vertical angle of a cone of height 3 cm is 60°, find its volume.
13. If a sphere of radius 6 cm is melted and drawn into a wire of diameter $\frac{1}{5}$ cm, find the length of the wire.
14. What is the radius of a cylinder whose volume and curved surface area are numerically equal?
15. What is the radius of a solid hemisphere whose volume and the whole surface area are numerically equal?
16. If the surface area of a sphere is 616 cm$^2$, what is its volume?
17. The radius of a solid hemispherical toy is 3.5 cm. What is its total surface area?
18. Two cubes each of volume 64 cm$^3$ are joined end to end. What is the surface area of the resulting cuboid?
19. What is the ratio of the volume of a cube to that of a sphere which will exactly fit inside the cube?
20. If height of a frustum is 4 cm and the radii of two bases 3 cm and 6 cm respectively, what is the slant height of the frustum?

PRACTICE TEST

M.M : 30

Time : 1 hour

General Instructions :

Q. 1-4 carry 2 marks, Q. 5-8 carry 3 marks and Q. 9-10 carry 5 marks each.

1. A cylindrical container has diameter 28 cm and contains sufficient water in it to submerge a cuboid 11 cm × 7 cm × 16 cm. Find the rise in level of the water when the cuboid is submerged in it.
2. If the number of square centimetres on the surface of a sphere is equal to the number of cubic centimetres in its volume, what is the diameter of the sphere?
3. The diameter of a metallic solid sphere is 12 cm. It is melted and drawn into a wire having diameter of the cross-section 0.2 cm. Find the length of the wire.
4. A heap of wheat is in the form of a cone of radius 4.5 m and height 3.5 m. How much canvas cloth is required to just cover the heap?
5. A hemispherical tank of radius 2.1 m is full of water. It is connected with a pipe which empties it at the rate of 7 litres per second. How much time will it take to empty the tank completely? (1 litre = 1000 cm$^3$)
6. A semi-circular thin sheet of metal of diameter 28 cm, is bent to make an open conical cap. Find the capacity of the cap.
7. A cylindrical jar of radius 6 cm contains oil. Iron spheres each of radius 1.5 cm are immersed in the oil. How many spheres are necessary to raise the level of oil by 2 cm?
8. A barrel of fountain pen, cylindrical in shape, is 7 cm long and 5 mm in radius. A full barrel of ink is used for writing 165 words on an average. How many words would use a bottle of ink containing 1/10th of a litre.
9. The interior of a building is in the form of a cylinder of diameter 4.3 m height 3.8 m, surmounted by a cone whose vertical angle is a right angle. Find the area of the surface and the volume of the building.
10. A solid right circular cone of base radius 6 cm and height 12 cm is made up of metal. A right circular cone of height 4 cm is removed from the original cone leaving a frustum as shown.

Find:
(i) the radius of the right circular cone which has been removed.
(ii) the volume of the frustum.
(iii) the whole surface area of the frustum.

ANSWERS OF PRACTICE EXERCISE

1. 350 cm²  2. 9735 m²  3. 266.11 cm³  4. 641.66 cm³, 418 cm³  5. 1221.94 Rs.
6. 218.064 cm³  7. Rs. 457.60  8. 1388.7 kg  9. 500 π m², Rs. 55000
10. 616 cm³  11. 770 cm²  12. 100  13. 892.26 kg  14. 44 m², Rs. 22000
15. 3 cm  16. 683.83 cm³  17. 72.35 m², 65.15 m³  18. 4 m
19. 124.4 cm³  20. 14850 m²  21. 506.88 kg  22. 502.07 cm³, 444.7 cm³  23. 3768 cm³, 1318.8 cm²
24. 0.9496 m³, 5.94 m²
25. Area to be painted orange = 63.585 cm², Area to be painted yellow = 195.465 cm²
26. 190.93 cm³  27. 179.66 cm³  28. 6 : π  29. 1000  30. 2.5 cm
31. 1500  32. 72  33. 2 cm  34. 3.5 cm  35. 1 cm  36. 1.6 cm
37. \( \frac{51}{77} \) cm  38. 2 cm  39. 0.5 cm  40. 1.75 cm  41. 50 cm  42. 33.6 cm
43. 1.215 m  44. 90  45. 54  46. 18 cm  47. \( \frac{8}{3} \) cm  48. 216
49. 1.5 m  50. 1.6 m (approx)  51. 3 cm, 12 cm  52. 126  53. 8
54. 3 : 8  55. 78.75 cm  56. 225000 m³  57. 9979.2 l  58. 26.73 minutes
59. 51 minutes 12 seconds  60. 10 hours  61. L.S.A = 628.57 cm², T.S.A. = 1357.71 cm²
62. 48510 cm³  63. 7599.42 cm²  64. L.S.A. = 817.14 cm², vol. =4324.57 cm³
65. Rs. 2143.05  66. height = 15 cm, Area = 2160.32 cm²
67. capacity = 20.02 l, amount of sheet required = 3068.8 cm², Total cost = Rs. 61.34
68. cost of milk = Rs. 167.20, cost of metal sheet used = Rs. 97.97  69. 74.26 cm²
70. 1 : 2\(^1/2\) − 1  71. 20 cm  72. 1 : 2  73. 1 : 7
74. (i) 12 cm  (ii) 312 π cm³  (iii) 129 π cm²

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ANSWERS OF MULTIPLE CHOICE QUESTIONS

1. (c) 2. (b) 3. (a) 4. (b) 5. (b)
6. (c) 7. (c) 8. (b) 9. (c) 10. (b)
11. (c) 12. (c) 13. (b) 14. (d) 15. (a)

ANSWERS OF VERY SHORT ANSWER TYPE QUESTIONS

1. 54 cm$^2$ 2. 12 cm 3. 1188 cm$^2$ 4. $\frac{343\pi}{6}$ cm$^3$ 5. 1 : 2 : 3
6. 3 cm 7. 20 cm 8. $12\sqrt{3}$ cm 9. 6 units 10. 30$\sqrt{2}$ cm
11. 13 cm 12. $27\pi$ cm$^3$ 13. 288 m 14. 2 units 15. 4.5 units
16. 1437.33 cm$^3$ 17. 115.5 cm$^2$ 18. 160 cm$^2$ 19. 6 : $\pi$
20. 5 cm

ANSWERS OF PRACTICE TEST

1. 2 cm 2. 6 cm 3. 288 m 4. 80.61 m$^2$ 5. 46.2 minutes
6. 622.38 cm$^3$ 7. 16 8. 3000 words 9. 71.828 m$^2$, 65.62 m$^3$ (approx)
10. (i) 2 cm (ii) 435.8 cm$^3$ (iii) 350.58 cm$^2$